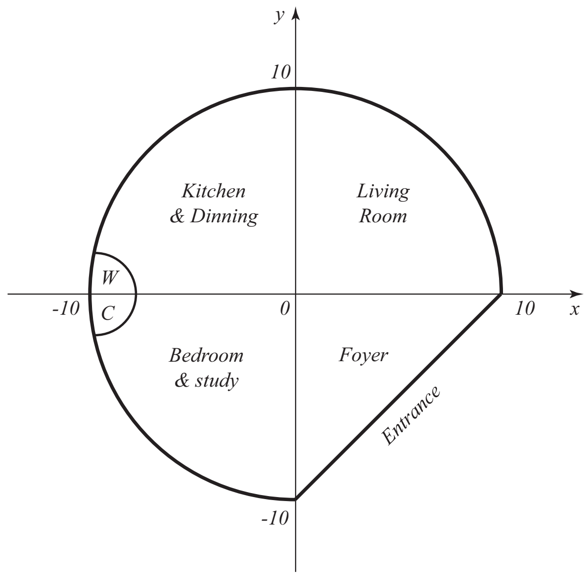
Math 10360 – Example Set 07A
Sections 14.1, 15.1, & 15.2
Two Variable Functions & Double Integrals

1. Trig Cossin the hobbit wanted to build a house with a circular floor plan with a cut-off for the entrance. The floor plan (top-view) for the house is shown below. The height of the house over each point on the floor (under the roof) is given by the function

$$f(x, y) = 40 - 2x + 2y \quad \text{meter.}$$

This means that the roof is $f(x, y)$ over the point (x, y) on the floor.



The house is divided more or less four regions; the triangular foyer, the quarter circular regions living room, kitchen and dinning, bedroom and study. All dimensions are in meters.

1a. Where are you standing in if you are at $(5, 2.5)$? at $(-5, -5)$? Find the height of the house at these points.

1b. Parametrized (label) the foyer region in terms of x and y. Give two ways of doing this.

1c. Use the ideas in Riemann sums to find the internal volume of the hobbit house over the foyer of the house. That is the volume enclosed by the roof over the foyer of the house.

Summary: The volume under the graph of a **positive-valued** function $z = f(x, y)$ over a region R in the xy-plane is given by

$$\iint_R f(x, y)dA$$

To evaluate the integral you need to parametrize the region R then integrate $f(x, y)$ with respect to x followed by y or y followed by x.

Math 10360 – Example Set 07B
Sections 15.1 & 15.2
Double Integral in Cartesian Coordinates

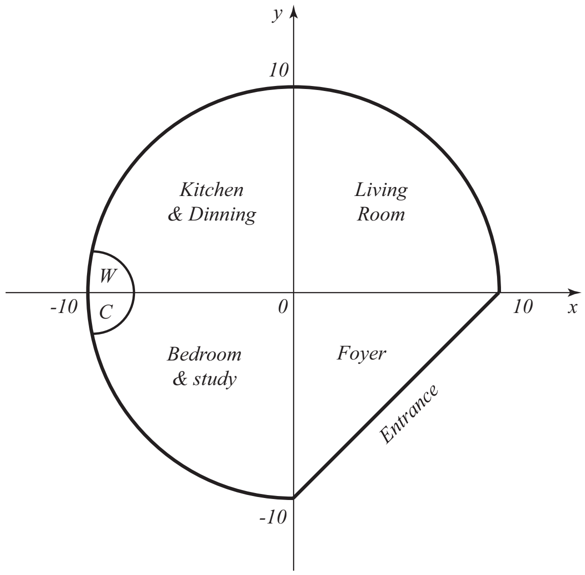
1. Give a sketch of the given region R in the xy-plane then evaluate the double integral

$$\int\int_R (6x^2y + xe^y)\, dA.$$

1a. $R = [-2, 2] \times [-1, 0]$. There are two ways which are just as easy. Do both.

1b. R is the region enclosed by $y = x^2$, $y = -x$, and $x = 2$.

Math 10360 – Example Set 07C
Section 15.4 Double Integral in Polar Coordinates



1. Referring to Trig Cossin's hobbit house, a floor mat of varying density $\rho(x, y) = 10 - 2xy$ kg/m^2 will be used to line the foyer region. Use the ideas in Riemann sum to compute the total mass of the mat.

Polar Coordinates. For circular regions centered at the origin, it is often simpler to parametrize (label) the region in terms of polar coordinates (r distance from the origin, and θ angle measured from the positive half of the x-axis) instead of the cartesian (or rectangular) coordinates (x, y).

The relations between the polar coordinates (r, θ) of a given on the plane and its cartesian coordinates (x, y) are given by

$$x = r\cos(\theta); \qquad y = r\sin(\theta)$$

As a convention, we may restrict $r > 0$ and $0 \le \theta < 2\pi$. Can you see the following the relations?

$$x^2 + y^2 = r^2; \qquad \tan\theta = \frac{y}{x}$$

2. Convert the following rectangular coordinates to polar coordinates: **(a)** $(3, -3)$ and **(b)** $(-2, -\sqrt{3})$.

3. Convert the following polar coordinates to rectangular coordinates: **(a)** $(\sqrt{2}, \pi/4)$ and **(b)** $(2, 7\pi/6)$.