Math 10360 – Example Set 11A

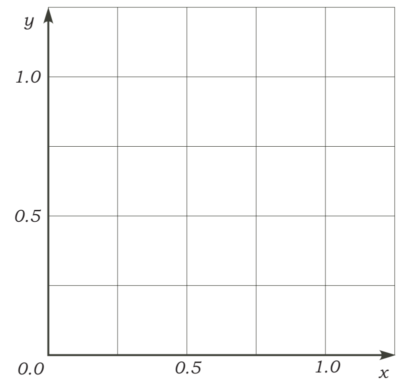
Topic: (9.3) Slope fields and the solutions of differential equations.

Consider the equation $y' = x^2 + y^2$

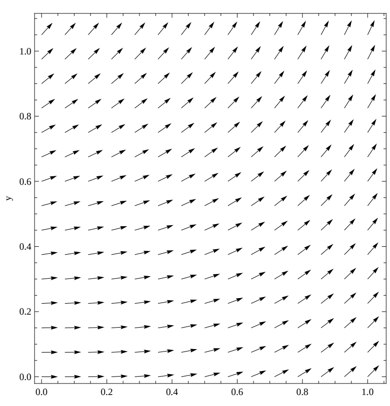
(A) Compute the slope of the solution curve $y(x)$ at the coordinate pairs given in the table below.

$y \backslash x$	0	0.5	1
0			
0.5			
1			

(B) Draw the slope fields at these points



(C) Below is a computer generated slope field for $y' = x^2 + y^2$. Use it to sketch the solution of the initial value problem $y' = x^2 + y^2, \quad y(0) = 0.5$



(D) Find the **linear approximation** at $x = 0$ for the solution of $y' = x^2 + y^2, \quad y(0) = 0.5$. Use it to estimate the value of $y(0.1)$.

(E) We could repeat the computation in (d) to estimate the value of $y(0.3)$. We call this method of estimation Euler's method with 3 equal steps of size $h = \Delta x = 0.1$.

Given: $y(0) = 0.5$

$y(0.1) \overset{LA}{\approx} y(0) + \boldsymbol{y'(0)}(0.1) = 0.5 + (0.25)(0.1) = 0.525$

$y(0.2) \overset{LA}{\approx}$ $= 0.554$

$y(0.3) \overset{LA}{\approx}$ $= 0.588.$

2. The weight w in kilograms of a kind of tropical fungus is modeled by the differential equation

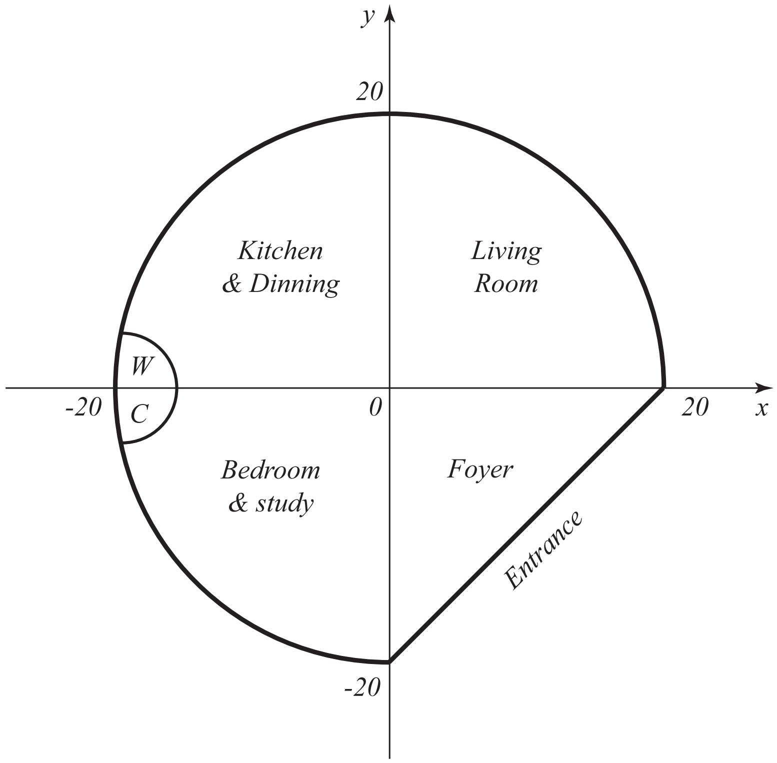
$$\frac{dw}{dt} = \frac{\sqrt{w}}{t^2 + 1}; \qquad w(1) = 4.$$

Here t denotes the time measured in weeks. Estimate the weight of the fungus at $t = 2.5$ weeks using Euler's method with three steps.

Math 10360 – Example Set 11B

Section 14.3 Partial Derivatives

1. Consider the height function of the hobbit house $f(x, y) = 14 - \frac{1}{100}(x^2 + y^2)$ over the given floor plan.



1a. When s hobbit climbs on the roof along $y = 10$, find how fast is his height $f(x, y)$ is changing with respect to x.

1b. When y is arbitrarily fixed, find how fast $f(x, y)$ is changing with respect to x.

1c. When $x = 1$, find how fast $f(x, y)$ is changing with respect to y.

1d. When x is arbitrarily fixed, find how fast $f(x, y)$ is changing with respect to y.

Definition (Partial Derivative). Let $f(x, y)$ a function of two variables. Then we define:

(A) the partial derivative of f with respect to x at the point (a, b) by the limit:

$$\frac{\partial f}{\partial x}(a, b) = f_x(a, b) = \lim_{\Delta x \to 0} \frac{f(a + \Delta x, b) - f(a, b)}{\Delta x}$$

This is the (instantaneous) rate of change of f in the $x-$direction at (a, b).

(B) the partial derivative of f with respect to y at the point (a, b) by the limit:

$$\frac{\partial f}{\partial y}(a, b) = f_y(a, b) = \lim_{\Delta y \to 0} \frac{f(a, b + \Delta y) - f(a, b)}{\Delta y}$$

This is the (instantaneous) rate of change of f in the $y-$direction at (a, b).

2. Evaluate the following limits:

2a. $\lim_{h \to 0} \frac{\ln(3x + 2(y+h)) - \ln(3x+2y)}{h}$

2b. $\lim_{h \to 0} \frac{\ln(3(x+h) + 2y) - \ln(3x+2y)}{h}$

3. Find all first and second partial derivatives of the function $g(x, y) = xe^{x^2y}$.

Math 10360 – Example Set 11C

Section 14.3 (Partial Derivatives): Estimating Partial Derivatives

1. The temperature (F) adjusted for wind-chill is a temperature which tells you how cold it feels, as a result of the combination of wind-chill (W) and temperature (T). So we have $F(T, W)$. (Source: Wikipedia)

**	**25**	−31	−24	−17	−11	−4	3	9	16	23
Wind	**20**	−29	−22	−15	−9	−2	4	11	17	24
Speed	**15**	−26	−19	−13	−7	0	6	13	19	25
(W mph)	**10**	−22	−16	−10	−4	3	9	15	21	27
	5	−16	−11	−5	1	7	13	19	25	31
**	**	−5	**0**	**5**	**10**	**15**	**20**	**25**	**30**	**35**

Temperature (T°F)

1a. Estimate the rate of change of F with respect to T at $T = 20$ and $W = 15$? In another words, estimate the rate of change of F in the $T-$direction at $(20, 15)$. You should compute as many estimates as the data allows.

1b. Estimate the rate of change of F with respect to W at $T = -5$ and $W = 25$? In another words, estimate the rate of change of F in the $W-$direction at $(-5, 25)$. You should compute as many estimates as the data allows.

1c. If temperature is fixed at $T = 20$, and windspeed increases from 15 mph to 15.8 mph, what is the estimated change in the temperature F adjusted for wind-chill.

Section 14.6 Chain Rule

2. Find formulas for the following derivatives by first drawing a tree diagram to connect all related quantities:

2a. $\dfrac{du}{dt}$ where $u = \ln(x^2 + y^2)$; $x = \cos 2t$ and $y = \sin t$.

2b. $\dfrac{\partial u}{\partial t}$ and $\dfrac{\partial u}{\partial s}$ where $u = e^{x_1 + 4x_2 - x_3}$; $x_1 = 2t - s$, $x_2 = t^2$ and $x_3 = t + 3s$.