Math 10360 – Example Set 11A

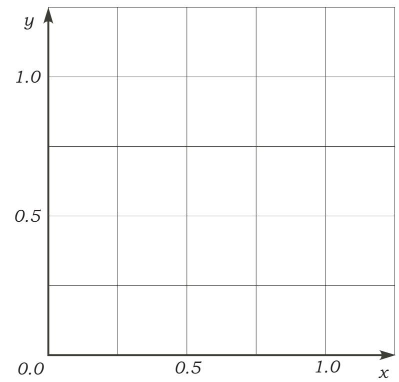
Topic: (9.3) Slope fields and the solutions of differential equations.

Consider the equation $y' = x^2 + y^2$

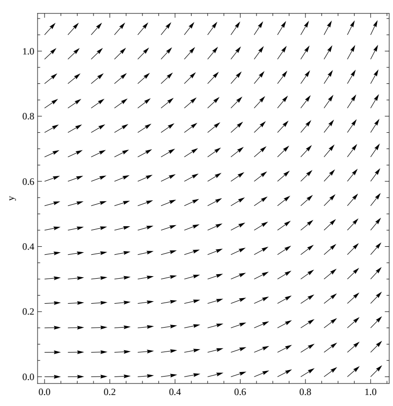
(A) Compute the slope of the solution curve $y(x)$ at the coordinate pairs given in the table below.

$y\backslash x$	0	0.5	1
0			
0.5			
1			

(B) Draw the slope fields at these points.



(C) Below is a computer generated slope field for $y' = x^2 + y^2$. Use it to sketch the solution of the initial value problem $y' = x^2 + y^2, \quad y(0) = 0.5$



(D) Find the **linear approximation** at $x = 0$ for the solution of $y' = x^2 + y^2, \quad y(0) = 0.5$. Use it to estimate the value of $y(0.1)$.

(E) We could repeat the computation in (D) to estimate the value of $y(0.3)$. We call this method of estimation Euler's method with 3 equal steps of size $h = \Delta x = 0.1$.

Given: $y(0) = 0.5$

$y(0.1) \overset{LA}{\approx} y(0) + \boldsymbol{y'(0)}(0.1) = 0.5 + (0.25)(0.1) = 0.525$

$y(0.2) \overset{LA}{\approx}$ ≈ 0.554

$y(0.3) \overset{LA}{\approx}$ $\approx 0.588.$

2. The weight w in kilograms of a kind of tropical fungus is modeled by the differential equation

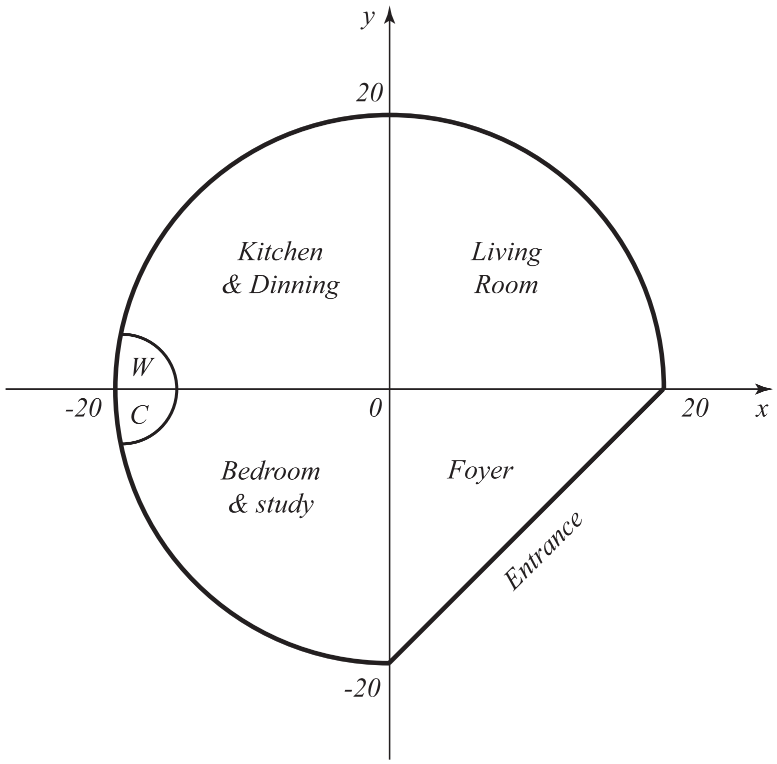
$$\frac{dw}{dt} = \frac{\sqrt{w}}{t^2+1}; \qquad w(1) = 4.$$

Here t denotes the time measured in weeks. Estimate the weight of the fungus at $t = 2.5$ weeks using Euler's method with three steps.

Math 10360 – Example Set 11B

Section 14.3 Partial Derivatives

1. Consider the height function of the hobbit house $f(x,y) = 14 - 10e^{-(x^2+y^2)}$ over the given floor plan.



1a. When hobbit Arc Tangy climbs on the roof along $y = 10$, find how fast is his height $f(x,y)$ is changing with respect to x (along the $y-$sectional curve)?

1b. When y is arbitrarily fixed, find how fast $f(x,y)$ is changing with respect to x?

1c. When $x = 5$, find how fast $f(x,y)$ is changing with respect to y (along the $x-$sectional curve)?

1d. When x is arbitrarily fixed, find how fast $f(x,y)$ is changing with respect to y?

Definition (Partial Derivative). Let $f(x,y)$ a function of two variables. Then we define:

(A) the partial derivative of f with respect to x at the point (a,b) by the limit:

$$\frac{\partial f}{\partial x}(a,b) = f_x(a,b) = \lim_{\Delta x \to 0} \frac{f(a+\Delta x, b) - f(a,b)}{\Delta x}$$

This is the (instantaneous) rate of change of f in the $x-$direction at (a,b).

(B) the partial derivative of f with respect to y at the point (a,b) by the limit:

$$\frac{\partial f}{\partial y}(a,b) = f_y(a,b) = \lim_{\Delta y \to 0} \frac{f(a, b+\Delta y) - f(a,b)}{\Delta y}$$

This is the (instantaneous) rate of change of f in the $y-$direction at (a,b).

2. Evaluate the following limits:

2a. $\lim_{h \to 0} \dfrac{\ln(3x + 2(y+h)) - \ln(3x + 2y)}{h}$

2b. $\lim_{h \to 0} \dfrac{\ln(3(x+h) + 2y) - \ln(3x + 2y)}{h}$

3. Find all first and second partial derivatives of the function $f(x, y) = 14 - 10e^{-(x^2+y^2)}$