Math 10360 – Example Set 12A

Section 14.3 (Partial Derivatives): Estimating Partial Derivatives

1. The temperature (F) adjusted for wind-chill is a temperature which tells you how cold it feels, as a result of the combination of wind-chill (W) and temperature (T). So we have $F(T, W)$. (Source: Wikipedia)

**	**25**	−31	−24	−17	−11	−4	3	9	16	23
Wind	**20**	−29	−22	−15	−9	−2	4	11	17	24
Speed	**15**	−26	−19	−13	−7	0	6	13	19	25
(W mph)	**10**	−22	−16	−10	−4	3	9	15	21	27
	5	−16	−11	−5	1	7	13	19	25	31
**	**	**−5**	**0**	**5**	**10**	**15**	**20**	**25**	**30**	**35**

Temperature (T°F)

1a. Estimate the rate of change of F with respect to T at $T = 20$ and $W = 15$? In another words, estimate the rate of change of F in the $T-$direction at $(20, 15)$. You should compute as many estimates as the data allows.

1b. Estimate the rate of change of F with respect to W at $T = -5$ and $W = 25$? In another words, estimate the rate of change of F in the $W-$direction at $(-5, 25)$. You should compute as many estimates as the data allows.

1c. If temperature is fixed at $T = 20$, and windspeed increases from 15 mph to 15.8 mph, what is the estimated change in the temperature F adjusted for wind-chill.

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Section 14.6: Chain Rule

Section 14.6 Chain Rule

2. Find formulas for the following derivatives by first drawing a tree diagram to connect all related quantities:

2a. $\dfrac{du}{dt}$ where $u = \ln(x^2 + y^2)$; $x = \cos 2t$ and $y = \sin t$.

2b. $\dfrac{\partial u}{\partial t}$ where $u = e^{x_1+4x_2-x_3}$; $x_1 = 2t - s$, $x_2 = t^2$ and $x_3 = t + 3s$.

2c. $\left.\dfrac{\partial u}{\partial s}\right|_{\substack{s=1 \\ t=-1}}$ where $u = e^{x_1+4x_2-x_3}$; $x_1 = 2t - s$, $x_2 = t^2$ and $x_3 = t + 3s$.

Section 14.6 (Chain Rule): Implicit Differentiation

2. Given that $ze^{x+2y} + z^2 - x - y = 0$. Find $z_x = \dfrac{\partial z}{\partial x}$ and $z_y(1, -1, 0)$.

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Section 14.6: Chain Rule

Topic: Applications of Chain Rule

Linear Approximation of Change in a Function. Section 14.4 (Pg 808).

a. Consider a particle moving from point (a, b) to point $(a+k, b+h)$. If the particle travel at a constant speed and the total duration of the motion is 1 second, find in terms of time t (in seconds), a formula for the position (x, y).

b. Consider a function $f(x, y)$ such that its first partial derivatives exist for all points near (a, b). If (x, y) is a point on the line segment in Q2(a), find a formula for the rate of change of f with respect to t.

c. For a small change in time Δt, let the corresponding change in x from a be Δx, the corresponding change in y from b be Δy, and Δf be the corresponding change in f from $f(a, b)$. Then we have

$$\frac{\Delta f}{\Delta t} \approx \left.\frac{df}{dt}\right|_{t=0}.$$

Prove that

$$\boxed{\Delta f \approx \frac{\partial f}{\partial x}(a,b) \cdot \Delta x + \frac{\partial f}{\partial y}(a,b) \cdot \Delta y}$$

where $\Delta f = f(a + \Delta x, b + \Delta y) - f(a, b)$. This boxed formula is called the Linear Approximation of change in f when (x, y) changes from (a, b) to $(a + \Delta x, b + \Delta y)$.

Alternately, for all (x, y) near (a, b) then $x = a + \Delta x$ and $y = b + \Delta y$ for some small Δx and Δy. Then we can estimate $f(x, y)$ by the formula:

$$\boxed{f(x,y) \approx f(a,b) + \frac{\partial f}{\partial x}(a,b) \cdot (x - a) + \frac{\partial f}{\partial y}(a,b) \cdot (y - b)}$$

We call the right hand side the linearization of $f(x, y)$ at (a, b).

1. Let $g(x, y) = \sqrt{4 - x^2 + y^2}$. Using linear approximation $g(x, y)$ at $(1, 1)$, estimate the following values:

(a) the change in $g(x, y)$ when (x, y) changes from $(1, 1)$ to $(1.1, 0.8)$.

(b) the value of $g(1.1, 0.8)$.

(c) the percentage change in $g(x, y)$ when (x, y) changes from $(1, 1)$ to $(1.1, 0.8)$.

Find also the linearization of $g(x, y)$ at $(1, 1)$.

(a) $\Delta g \approx -0.15$; (b) $g(1.1, 0.8) \approx 1.85$. Compare with calculator $g(1.1, 0.8) \approx 1.852025918$; (c) $\frac{\Delta g}{g(1,1)} \times 100\% \approx -3.75\%$; $g(x, y) \approx 2 - \frac{1}{2}(x - 1) + \frac{1}{2}(y - 1)$.