Math 10360 – Example Set 13A

Geometric Sequence and Series

Definition (Geometric Series): We called an infinite sum (series) of the form

$$c + cr + cr^2 + cr^3 + \cdots + cr^{n-1} + \cdots$$

a geometric series with first term c and common ratio r.

(Finite Sums of a Geometric Series): The sum formula for the first N terms of a geometric series:

$$c + \ cr \ + \ cr^2 \ + \ cr^3 \ + \ \cdots \ + \ cr^{N-1} = \frac{c(1 - r^N)}{1 - r}$$

Proof: Consider first the finite sum $S_N = c + cr + cr^2 + \cdots + cr^{N-1}$

$$S_N \ = \ c \ + \ cr \ + \ cr^2 \ + \ cr^3 \ + \ \cdots \ + \ cr^{N-1}$$

$$rS_N \ = \qquad cr \ + \ cr^2 \ + \ cr^3 \ + \ \cdots \ + \ cr^{N-1} \ + \ cr^N$$

$S_N - rS_N \stackrel{?}{=}$

So $(1 - r)S_N \stackrel{?}{=}$ $\Rightarrow \ S_N \stackrel{?}{=}$

Take limit $N \to \infty$ of the **finite sum** of a geometric series to find the sum (to infinity) of a geometric series with first term c and common ratio r.

Summary: Sum formulas for Geometric Series:

Consider a geometric series with first term c and common ratio r.

The sum formula for the first N terms of a geometric series:

$$c + cr + cr^2 + cr^3 + \cdots + cr^{N-1} = \underline{\qquad\qquad\qquad\qquad}$$

If $|r| < 1$ then the geometric series is **convergent** is sum is given by

$$c + cr + cr^2 + cr^3 + \cdots + cr^{n-1} + \cdots = \underline{\qquad\qquad\qquad\qquad}$$

If $|r| > 1$, then the geometric series is divergent.

1. What are the common ratio and first term of the geometric series $\displaystyle\sum_{n=3}^{\infty} \frac{2^{2n}}{3^n}$. What is the 20th partial sum? What is the sum (to infinity) of the series.

Math 10360 – Example Set 13B

Geometric Series and its Applications

2. Rewrite each of the following repeated decimals as a fraction.

a. $0.\bar{9} = 0.999\cdots \stackrel{?}{=}$

b. $3.0\,\overline{12} = 3.0121212\cdots \stackrel{?}{=}$

3. A drug is designed so that 60% remains in the body at the end of each 24 hour period (one day). If 30 mg of the drug is given daily to a patient find (A) the amount of drug in the body after 10 days before the next dose is given, and (B) the approximate amount of drug in the body after a very long time assuming measurement is done **before** the next dose is given.

4. A ball is **projected** from the ground to a height of 10 feet and allowed to freely fall and rebound when it hits the ground. If the ball rebounds to 80% of the height it fell from and is allowed to continue its motion indefinitely answer the questions below. If the bouncing persists, write down using summation notation the total distance travelled by the ball. What is the total vertical distance travelled by the ball?

5. Consider the sequence $\{c_n\}_{n=1}^{\infty}$ given by the iterative formula $c_{n+1} = (n+1)c_n$ for $n \geq 1$ and $c_1 = 1$. (a) Is the sequence $\{c_n\}$ geometric? (b) Find a formula for the general term c_n.