

**Math 10360 – Example Set 13A**  
**Section 14.6: Chain Rule**  
**Section 14.8: Linear Approximation**

**Sensitivity and Elasticity:** Let  $z = f(x, y)$ . Set  $x = a, y = b$ . Then the sensitivity of the quantity  $z$  relative to  $x$  at  $(a, b)$  is measured by

$$\frac{\partial f}{\partial x}(a, b).$$

We call this the **sensitivity coefficient** of  $f$  with respect to  $x$  at  $(a, b)$ .

Likewise, the sensitivity coefficient of  $f$  with respect to  $y$  at  $(a, b)$  is  $\frac{\partial f}{\partial y}(a, b)$ .

On the other hand, the elasticity of the quantity  $z$  relative to  $x$  is the percentage change in  $z$  given a 1% change in  $x$  from  $x = a$  (with no change in  $y = b$ ). This percentage change in  $z$  is also called **elasticity coefficient** with respect to  $x$  at  $(a, b)$ . Likewise, the elasticity of the quantity  $z$  relative to  $y$  is the percentage change in  $z$  given a 1% change in  $y$  from  $y = b$  (with no change in  $x = a$ ). This percentage change in  $z$  is also called elasticity coefficient with respect to  $y$  at  $(a, b)$ .

Let  $z = f(x_1, x_2, \dots, x_n)$ . Set  $x_1 = a_1, x_2 = a_2, \dots, x_n = a_n$ . Then the sensitivity of the quantity  $z$  relative to  $x_i$  at  $(a_1, a_2, \dots, a_n)$  is measured by

$$\frac{\partial f}{\partial x_i}(a_1, a_2, \dots, a_n).$$

We call this the **sensitivity coefficient** of  $f$  with respect to  $x_i$ .

On the other hand, the **elasticity** of the quantity  $z$  relative to  $x_i$  is the **linear approximation** of the percentage change in  $z$  given a 1% change in  $x_i$  from  $x_i = a_i$  (with no change in the other variable). This **estimated** percentage change in  $z$  is also called **elasticity coefficient** with respect to  $x_i$  at  $(a_1, a_2, \dots, a_n)$ . In general the elasticity coefficients of  $z$  are dependent on the independent values  $x_1, x_2, \dots, x_n$ .

**1.** Consider a cylindrical rod with height (length) 100cm and diameter 5cm.

**1a.** If the measuring instrument has an error of 0.1 cm, estimate using linear approximation the corresponding (propagated) error in the value of the volume if the above measurements are used.

**1b.** What is sensitivity of the volume of the given rod to its (i) height and (ii) diameter? That is compute the sensitivity coefficients (first partial derivatives) with respect to the height and (independently) the diameter at  $(100, 5)$ .

**1c.** Discuss the elasticity (proportional sensitivity) of the volume relative to its dimensions. That is discuss how the percentage change in volume for a 1% change in the height (alone) compared to the percentage change in volume for a 1% change in the diameter (alone).

Math 10360 – Example Set 13B  
Section 10.1 Sequences  
Section 10.2 Summing an Infinite Series

**Introduction to Sequences and Series.** A sequence (of numbers) is an ordered listing of numbers.

1. For each sequence below, write down (A) the formula for the general term of the sequence example and (B) limit of the sequence.

1a.  $\frac{2}{4 \cdot 5}, \frac{2}{5 \cdot 6}, \frac{2}{6 \cdot 7}, \frac{2}{7 \cdot 8}, \dots$

(A) General term:  $a_n =$  \_\_\_\_\_ for  $n = 1, 2, 3, \dots$

(B) Limit of the sequence  $\{a_n\}_{n=1}^{\infty}$  is the value  $a_n$  approaches as  $n$  gets larger and larger unboundedly. Find the limit of the sequence  $\{a_n\}_{n=1}^{\infty}$ .

Notation:  $\lim_{n \rightarrow \infty} a_n \stackrel{?}{=}$

1b.            2,                    -2,                    2,                    -2,    ...

(A) General term:  $b_n =$  \_\_\_\_\_ for  $n = 1, 2, 3, \dots$

(B) Find the limit of the sequence  $\{b_n\}_{n=1}^{\infty}$ .

Notation:  $\lim_{n \rightarrow \infty} b_n \stackrel{?}{=}$

2. The sum of a sequence is called a **series**. For example adding the terms of the sequence  $\{a_n\}_{n=1}^{\infty}$  in Q1a above gives the series:

$$\sum_{n=1}^{\infty} a_n \stackrel{?}{=}$$

Find the sum (value) of the above series using the following steps

a. Find  $S_N$  the  $N$ th partial sum of the given series.

b. Find the value of the given series by taking the limit of its  $N$ th partial sum. Is the given series convergent?

3a. Find the sum of the first 51 terms in the series  $\sum_{n=3}^{\infty} (\sqrt{n+1} - \sqrt{n})$ .

3b. Is the series  $\sum_{n=3}^{\infty} (\sqrt{n+1} - \sqrt{n})$  convergent?

**Summary.** The sum (value) of a series  $\sum_{n=1}^{\infty} a_n$  is found using

The **Nth partial sum** of the series  $\sum_{n=1}^{\infty} a_n$  is the sum  $S_N$  of the first  $N$ th terms of the series that is

the value:  $S_N =$  \_\_\_\_\_

The sum of a series  $\sum_{n=1}^{\infty} a_n$  is defined to be the limit of the  $N$ th partial sum  $S_N$ . That is the value:

$$\sum_{n=1}^{\infty} a_n = \lim_{N \rightarrow \infty} \left( \text{_____} \right) = \lim_{N \rightarrow \infty} S_N$$

(a) If  $\lim_{N \rightarrow \infty} S_N = S$  then we say that the series converges to  $S$  and write  $\sum_{n=1}^{\infty} a_n = S$  (the sum of the infinite series).

(b) If  $\lim_{N \rightarrow \infty} S_N$  does not exist then we say that the series diverges and the sum of the infinite series  $\sum_{n=1}^{\infty} a_n$  does not exist.

Math 10360 – Example Set 13C  
Section 10.1 Sequences  
Section 10.2 Summing an Infinite Series

**Geometric Sequence and Series**

**Definition (Geometric Sequence):** A sequence  $\{c_n\}_{n=1}^{\infty}$  is said to be geometric if the ratio of its consecutive terms  $\frac{c_{n+1}}{c_n}$  is a \_\_\_\_\_.

We call such a sequence a **geometric sequence** with first term  $c_1 = c$  and common ratio  $r$ . We list out the first four terms of the geometric sequence:

The  $N$ th term  $c_N$  of the sequence is given by \_\_\_\_\_.

**Definition (Geometric Series):** We called an infinite sum (series) of the form

$$c + cr + cr^2 + cr^3 + \dots + cr^{n-1} + \dots$$

a geometric series with first term  $c$  and common ratio  $r$ .

**(Finite Sums of a Geometric Series):** The sum formula for the first  $N$  terms of a geometric series:

$$c + cr + cr^2 + cr^3 + \dots + cr^{N-1} = \frac{c(1 - r^N)}{1 - r}$$

**Proof:** Consider first the finite sum  $S_N = c + cr + cr^2 + \dots + cr^{N-1}$

$$S_N = c + cr + cr^2 + cr^3 + \dots + cr^{N-1}$$

$$rS_N = cr + cr^2 + cr^3 + \dots + cr^{N-1} + cr^N$$

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$$S_N - rS_N \stackrel{?}{=}$$

$$\text{So } (1 - r)S_N \stackrel{?}{=} \Rightarrow S_N \stackrel{?}{=}$$

Take limit  $N \rightarrow \infty$  of the **finite sum** of a geometric series to find the sum (to infinity) of a geometric series with first term  $c$  and common ratio  $r$ .

## Summary: Sum formulas for Geometric Series:

Consider a geometric series with first term  $c$  and common ratio  $r$ .

The sum formula for the first  $N$  terms of a geometric series:

$$c + cr + cr^2 + cr^3 + \dots + cr^{N-1} = \underline{\hspace{10em}}$$

If  $|r| < 1$  then the geometric series is **convergent** is sum is given by

$$c + cr + cr^2 + cr^3 + \dots + cr^{n-1} + \dots = \underline{\hspace{10em}}$$

If  $|r| \geq 1$ , then the geometric series is divergent.

1. What are the common ratio and first term of the geometric series  $\sum_{n=3}^{\infty} \frac{2^{2n}}{3^n}$ . What is the 20th partial sum? What is the sum (to infinity) of the series.

## Geometric Series and its Applications

2. Rewrite each of the following repeated decimals as a fraction.

a.  $0.\bar{9} = 0.999\dots \stackrel{?}{=}$

b.  $3.0\bar{12} = 3.0121212\dots \stackrel{?}{=}$