Math 10360 – Example Set 14A

Testing Convergence of General Series.

Recall that a **geometric series** $\sum c_n$ with common ratio $\dfrac{c_{n+1}}{c_n} = r$. Then we have:

(a) $\sum c_n$ converges if ________________________.

(b) $\sum c_n$ diverges if ________________________.

For general series which are **NOT** geometric we can apply the Ratio Test.

Ratio Test Let $\sum a_n$ be a series with no zero terms. Consider the value ρ given by

$$\rho = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| .$$

Then we have the following:

(a) $\sum a_n$ converges (in fact, absolutely) if ________________________.

(b) $\sum a_n$ diverges if ________________________ or ________________________.

(c) The Ratio Test is inconclusive if ________________________.

1. Determine if the following series are convergent or divergent.

a. $\displaystyle\sum_{n=1}^{\infty} \frac{(-7)^n}{n^5}$ **b.** $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}$ **c.** $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+2}$

Introduction to Power Series

A **Power Series** can be thought about as a polynomial with infinitely many terms or arbitrarily high degree. Here are some examples:

(1) $$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots$$

(2) $$\sum_{k=1}^{\infty} \frac{1}{k+1}(x-2)^k = \frac{1}{2}(x-2) + \frac{1}{3}(x-2)^2 + \frac{1}{3}(x-2)^3 + \cdots + \frac{1}{n+1}(x-2)^n + \cdots$$

A general power series has the form:

$$a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + \cdots + a_n(x-c)^n + \cdots$$

where the coefficients $a_0, a_1, a_2, a_3, ...$ is a sequence of real numbers.

We call this a power series centered at $x = c$. Fill in the blanks below.

(1) $$\sum_{k=0}^{\infty} x^k = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots$$

(1) is called a power series centered at $x =$ ________.

(2) $$\sum_{k=1}^{\infty} \frac{1}{k+1}(x-2)^k = \frac{1}{2}(x-2) + \frac{1}{3}(x-2)^2 + \frac{1}{3}(x-2)^3 + \cdots + \frac{1}{n+1}(x-2)^n + \cdots$$

(2) is called a power series centered at $x =$ ________.

2. Find the values of x for which each of the following power series is convergent. You may ignore the discussion if the power series is convergent at the end-points of the interval found. What is the radius of convergent?

a. $\displaystyle\sum_{k=1}^{\infty} \frac{(x-2)^k}{k^3}$ **b.** $\displaystyle\sum_{k=1}^{\infty} \frac{x^{2k}}{2k+1}$ **c.** $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$