Math 10360 Example Set 15A
Sections 10.7 & 10.8 Taylor Polynomials & Taylor Series

Application of Taylor polynomial and Taylor Series

1a. Find the Talyor Series centered at $x = 0$ for $f(x) = e^x$. What is the interval of convergence for this power series?

1b. Using the Maclaurin polynomial $T_4(x)$ for e^x, estimate $e^{0.2}$.

1c. Write down the error of your estimate for $e^{0.2}$ in Q2(b) as a series. Explain your answer.

1d. Estimate the value of $\int_0^{0.2} e^{-x^2}\,dx$ using $T_4(x)$ for e^x.

2a. Find the 3rd-degree Taylor polynomial of $y(t)$ centered at zero, where $y(t)$ is the solution of the initial value problem

$$y' = y^2 + ty, \quad y(0) = -1$$

Use your result to estimate $y(0.3)$.

2b. Find the 3rd-degree Taylor polynomial of $y(t)$ centered at 1, where $y(t)$ is the solution of the initial value problem

$$y' = y^2 + ty, \quad y(1) = -1$$

Use your result to estimate $y(0.8)$.

3. Using the Taylor series for $\dfrac{1}{1+x^2}$ centered at $x = 0$ and differentiation, find the Maclaurin series for $\dfrac{2x}{(1+x^2)^2}$.

4a. Using the Taylor series for $\dfrac{1}{1+x^2}$ centered at $x = 0$ and integration, show that the Taylor series for $\arctan x$ at 0 is:

$$\arctan x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{2k+1} \quad \text{for} \quad -1 < x < 1$$

4b. Write down the 7th Taylor polynomial for $\arctan x$ at 0. Estimate the value of $\arctan(0.5)$. Write down the error for the estimate you found as an infinite series using summation notation.

4c. Write down the Taylor series for $f(x) = \arctan(1 - x)$ centered at 1, giving the values of x for which the series is convergent.

5. The 3rd Maclaurin Polynomial of $f(x)$ is given by $\quad T_3(x) = 1 - x^2 + 4x^3$.

a. Find the values $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$. What could you say about the point $(0, f(0))$ on the graph of $f(x)$?

b. Find the Taylor polynomial of $g(x) = e^{f(x)}$ centered at $x = 0$ of degree 2.