
There are generally three ways to estimate the derivative of a function \( f(x) \) at \( x = a \).

All three ways uses:

(a) points (or values \( \{b\} \)) \( x \) near to \( a \).

(b) secant lines (chords) estimate the tangent line to \( f(x) \) at \( x = a \).

(1) **Forward Difference Estimate.**

If \( B \) is close to \( A \), then the secant line \( AB \) is almost parallel to the tangent line at \( x = a \).

So \( f'(a) \approx \frac{f(b) - f(a)}{b - a} \)

slope at \( x = a \)

slope of the secant \( AB \).
This estimate is called the forward difference estimate since \( b \) is \( > a \). If we write \( b = a + h \),

Then \( f'(a) \approx \frac{f(b) - f(a)}{b - a} \)

\[ = \frac{f(a+h) - f(a)}{a+h - a} \]

so \( f'(a) \approx \frac{f(a+h) - f(a)}{h} \)

\[ \overbrace{\text{forward difference formula}} \]
(2) \textbf{Backward Difference Estimate}

If \( C \) is close to \( A \) then the secant line \( AC \) is almost parallel to the tangent line of \( f(x) \) at \( x = a \).

\[
\text{So } f'(a) \approx \frac{f(a) - f(c)}{a - c}
\]

\[\text{slope at } \ x = a \quad \text{slope of the secant line } AC.\]

This estimate is called the backward difference estimate since \( C \) is \( < A \).

If we write \( C = a - h \), then

\[
\text{then } f'(a) \approx \frac{f(a) - f(a-h)}{a - (a-h)}
\]

\[
= \frac{f(a) - f(a-h)}{a - (a-h)}
\]

\[
\text{So } f'(a) \approx \frac{f(a) - f(a-h)}{h}
\]

\[\text{backward difference formula}\]
Central Difference Estimates.

If both \( B \) and \( C \) are close to \( A \) then the slope of the tangent line at \( x = a \) is almost the slope of chord \( BC \).

So \( f'(a) \approx \frac{f(b) - f(c)}{b - c} \)

This estimate is called the central difference estimate since \( a \) is between \( b \) and \( c \).

If \( a \) is exactly at the midpoint between \( b \) and \( c \),

Then \( b = a + h \) and \( c = a - h \).
Then we have:

\[ f'(a) \approx \frac{f(b) - f(c)}{b - c} \]

\[ = \frac{f(a+h) - f(a-h)}{a+h - (a-h)} \]

\[ = \frac{f(a+h) - f(a-h)}{a+h - a - h} \]

\[ f'(a) \approx \frac{f(a+h) - f(a-h)}{2h} \]

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central difference formula

This only works if both \( b \neq c \)

are equidistance from \( a \)

If not we just use the slope of chord BC at the beginning.