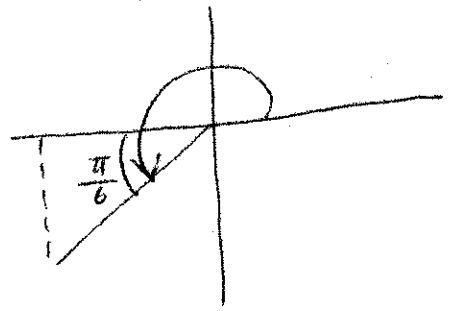


2(b)  $(2, 7\pi/6) \sim$  polar



$$x = 2 \cos \frac{7\pi}{6} = 2 \cdot \frac{-\sqrt{3}}{2} = -\sqrt{3}$$

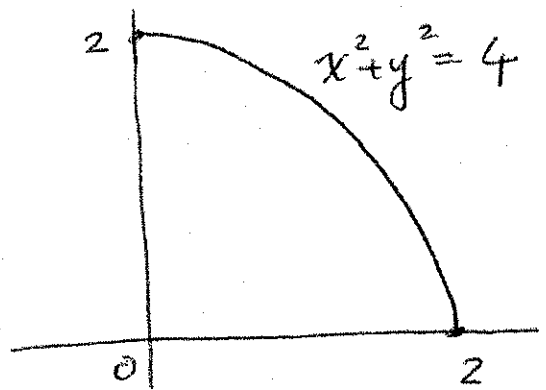
$$y = 2 \sin \frac{7\pi}{6} = 2 \cdot \frac{-1}{2} = -1$$

$(-\sqrt{3}, -1) \sim$  rectangular

Q3

Q:  $0 \leq r \leq 2$   
 $0 \leq \theta \leq \pi/2$

in polar coordinates



$$f(x, y) = 2x^2 + y^2 = x^2 + x^2 + y^2$$

$$f(r, \theta) = r^2 \cos^2 \theta + r^2 \leftarrow \text{density in polar coordinates}$$

$$\Delta M = f(r, \theta) \cdot \frac{\Delta A}{r \Delta \theta \Delta r} = (r^2 \cos^2 \theta + r^2) \cdot r \cdot \Delta \theta \Delta r$$

$$= (r^3 \cos^2 \theta + r^3) \Delta \theta \Delta r$$

$$M = \iint_Q f(r, \theta) \cdot dA = \int_0^2 \int_0^{\pi/2} (r^2 \cos^2 \theta + r^2) r d\theta dr$$

(2)

$$M = \int_0^2 \int_0^{\pi/2} (r^3 \cos^2 \theta + r^3) d\theta dr$$

$$= \int_0^2 \int_0^{\pi/2} \left[ r^3 \left( \frac{1 + \cos 2\theta}{2} \right) + r^3 \right] d\theta dr$$

$$= \int_0^2 \left[ r^3 \left( \frac{\theta + \frac{1}{2} \sin 2\theta}{2} \right) + r^3 \theta \right]_0^{\pi/2} dr$$

$$= \int_0^2 \left[ r^3 \left( \underbrace{\frac{\frac{\pi}{2} + 0}{2}}_{\frac{\pi}{4}} + r^3 \cdot \frac{\pi}{2} - 0 \right) \right] dr$$

$$= \int_0^2 \frac{3\pi}{4} r^3 dr = \frac{3\pi r^4}{16} \Big|_0^2 = \boxed{3\pi} \text{ kg}$$

You can also do:

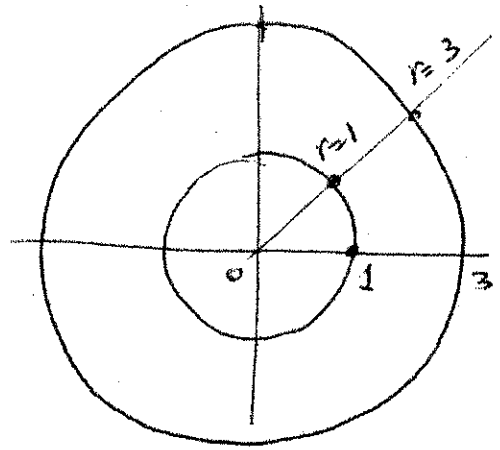
$$\int_0^{\pi/2} \int_0^2 f(r, \theta) \cdot r dr d\theta$$

(3)

Q4

$$R: 1 \leq r \leq 3$$

$$0 \leq \theta \leq 2\pi$$



$$\Delta P = \underbrace{\rho(r, \theta)}_{\text{density}} \cdot \underbrace{\Delta A}_{\text{area}}$$

$$= \frac{8000 \sin^2(\theta/2)}{9+r^2} \cdot \Delta A$$

$$P = \iint_R \rho(r, \theta) dA = \int_{r=1}^3 \int_{\theta=0}^{2\pi} \frac{8000 \sin^2(\theta/2)}{9+r^2} \cdot r d\theta dr$$

$$= \int_1^3 \int_0^{2\pi} \frac{8000r}{9+r^2} \left( \frac{1-\cos\theta}{2} \right) d\theta dr$$

$\underbrace{\frac{1-\cos\theta}{2}}_{\sin^2(\theta/2)}$

$$u = 9+r^2$$
$$du = 2r dr$$

$$= \int_1^3 \left[ \frac{8000r}{9+r^2} \left( \frac{\theta - \sin\theta}{2} \right) \right]_0^{2\pi} dr$$

$$= \int_1^3 \frac{8000r \cdot \pi}{9+r^2} dr = \int_{10}^{18} \frac{4000 \cdot 8000\pi}{u} \cdot \frac{1}{2} du$$

(4)

$$= 4000\pi [\ln 18 - \ln 10] = 4000\pi \ln(1.8)$$