Method of Substitution

1a. $\displaystyle\int_{-1}^{0} \frac{2x^3+1}{x^4+2x-1}\,dx$ $\qquad u = x^4+2x-1$

definite integral (a number after evaluation)

$$= \int_{x=-1}^{x=0} \frac{1}{x^4+2x-1}\,\underbrace{(2x^3+1)\,dx}$$

$$= \int_{-2}^{-1} \frac{1}{u}\cdot\frac{1}{2}\,du$$

u

$$= \int_{-2}^{-1} \frac{1}{2u}\,du$$

$u = x^4+2x-1$

$du = (4x^3+2)\,dx$

$= 2(2x^3+1)\,dx$

appears in the integrand

$\cancel{2}(2x^3+1)\,dx$

$= \frac{1}{2}\,du$

$u = x^4+2x-1$

$x=-1$:

$u = 1-2-1$

$= -2$

$x = 0$:

$u = 0+0-1$

$= -1$

16. $\int_{-1}^{0} \frac{2x^3+1}{x^4+2x-1}\,dx = \int_{-2}^{-1} \frac{1}{2u}\,du$

must simplier. (could be evaluated)

$= \left[\frac{1}{2}\ln|u|\right]_{-2}^{-1}$

important to include absolute value

do not switch u back to x

$= \frac{1}{2}\ln|-1| - \frac{1}{2}\ln|-2|$

$= \underbrace{\frac{1}{2}\ln(1)}_{0} - \frac{1}{2}\ln(2)$

$= -\frac{1}{2}\ln(2)$ #

2a. $\int \frac{6}{(3x-2)^5}\,dx$ ← indefinite integral

Expect $F(x)+C$ answer type.

$u = 3x-2$

$du = 3\,dx$

$dx = \frac{1}{3}\,du$

$$= \int \frac{\cancel{6}^{\,2}}{u^5}\cdot\frac{1}{\cancel{3}}\,du$$

$$= \int \frac{2}{u^5}\,du$$

$$= \int 2u^{-5}\,du$$

$$= \frac{2u^{-4}}{-4} + C$$

$$= -\frac{1}{2}u^{-4} + C$$ ← switch u to x

$$= -\frac{1}{2}(3x-2)^{-4} + C$$

OR

$$= -\frac{1}{2(3x-2)^4} + C$$

26. $\int \frac{6x}{(3x-2)^5}\,dx$ ← indefinite integral (expect $F(x) + C$)

$= \int \frac{6}{(3x-2)^5} \cdot x \cdot dx$

??

$u = 3x - 2$

$du = 3dx$

$dx = \frac{1}{3}du$

$u + 2 = 3x$

$x = \frac{u+2}{3}$

$= \int \frac{\overset{2}{\cancel{6}}}{u^5} \cdot \frac{u+2}{\cancel{3}} \cdot \frac{1}{3}\,du$

$= \frac{1}{3}\int \frac{u+2}{u^5}\,du = \frac{1}{3}\int\left(\frac{u}{u^5} + \frac{2}{u^5}\right)du$

$= \frac{1}{3}\int\left(u^{-4} + 2u^{-5}\right)du$

$= \frac{1}{3}\left[\frac{u^{-3}}{-3} + \frac{2u^{-4}}{-4}\right] + C$

$= \frac{1}{3}\left[-\frac{1}{3u^3} - \frac{1}{2u^4}\right] + C$

$= -\frac{1}{9u^3} - \frac{1}{6u^4} + C$

$= \underbrace{-\frac{1}{9(3x-2)^3} - \frac{1}{6(3x-2)^4}}_{\text{involves } x} + C$

3a. $\displaystyle\int_1^e \frac{\ln x}{x}\,dx$ ← ~~in~~ definite integral (expect a number)

$\displaystyle = \int_{\underline{1}}^{\underline{e}} \ln x \cdot \frac{1}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

appear in the integrand

$\displaystyle = \int_{\underline{0}}^{\underline{1}} u\,du$

no switching to x

$x = 1$:

$u = \ln 1 = 0$

$x = e$:

$u = \ln e = 1$

$= \left[\frac{u^2}{2}\right]_0^1$

$= \frac{1}{2} - \frac{0}{2}$

$= \frac{1}{2}$ ← expect a number

3b. $\int_{\pi/6}^{\pi/4} \sin^3 x \cos x \, dx$ ← definite integral (expect a number)

$$= \int_{\pi/6}^{\pi/4} (\underline{\sin x})^3 \underbrace{\cos x \, dx}_{} \quad {}^*$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int_{1/2}^{1/\sqrt{2}} u^3 \, du \quad {}^* \text{ kept}$$

$x = \pi/6$:

$u = \sin \pi/6 = 1/2$

$x = \pi/4$:

$u = \sin \pi/4 = \frac{1}{\sqrt{2}}$

$$= \left[\frac{u^4}{4} \right]_{1/2}^{1/\sqrt{2}}$$

$$= \frac{1}{4}\left(\frac{1}{\sqrt{2}}\right)^4 - \frac{1}{4}\left(\frac{1}{2}\right)^4$$

$(\sqrt{2})^4$

$= (2^{\frac{1}{2}})^4$

$= 2^2 = 4$

$$= \underbrace{\frac{1}{4}\left(\frac{1}{4}\right)}_{\frac{1}{16}} - \frac{1}{4}\left(\frac{1}{16}\right)$$

$$= \frac{1}{16}\left(1 - \frac{1}{4}\right)$$

$$= \frac{1}{16}\left(\frac{3}{4}\right) = \frac{3}{64}$$