Mathematics 20480
Introduction to Dynamical System for Scientists
Spring Semester 2025

Section	Instructor	Class Schedule	Office Hr	**email**@nd.edu
1	Arthur Lim	MWF 11:30 - 12:20 HAYE 127	See Website	arthurlim

Course Reference: The course content is a mix of linear algebra and its application in the context of solution of (linear) difference and (linear and non-linear) differential equations. There will also be substantial discussion of examples in Ecology, Physics, Epidemiology, etc. The self-contained course notes will be provided. But here are some references for the class below. You are NOT required to purchase them:

Linear Algebra and Its Applications (5th Edition) by D. Lay, S. Lay, and J. McDonald

Elementary Differential Equations and Boundary Value Problems (10th Edition) by W. Boyce and R. DiPrima

3,000 Solved Problems in Linear Algebra by S. Lipschutz

Schaum's Outline of Linear Algebra (5th Edition) by S. Lipschutz and M. Lipson

Course Website: http://www.nd.edu/∼m20480
Most information for this course are posted on its website. These include office hours and contact information, daily homework information, exam dates and venue, practice exams, and etc.

Calculator Policy: Calculators are allowed exams. You may use your calculators for homework and assignment, but it is strongly recommended that you do not rely on any of the special functions or programs on the calculator.

Course Grade & Breakdown:

	Date	Day	Time	Room	Points
Midterm 1	Feb. 20	Thursday	8:00am – 9:15am	TBA	100
Midterm 2	Mar 27	Thursday	8:00am – 9:15am	TBA	100
Midterm 3	Apr 17	Thursday	8:00am – 9:15am	TBA	100
Final	May 09	Friday	10:30am – 12:30pm	TBA	150
Online Hwk & Quizzes	Hwk maybe online or written. Quizzes TBA				100
Project	Team project				50
				Total points:	600

Cutoffs for major grades (A, B, C, D, F) for each exam will be assigned and announced in class so you have some indication of your level of performance. Your final grade will be based on your total score out of 600.

Missed exams: Note that there will be three Midterm Exams and a Final Exam. A student who misses an examination will receive **zero points** for that exam unless he or she has written permission from the Dean of your college. Please be aware that travel plans, sleeping in, defective alarm clocks, etc. are **not** considered to be a valid excuse! If you have a valid excuse (illness, excused athletic absence, etc.) for missing an exam, please see me ASAP (preferably before the exam) to schedule a makeup exam.

Exam conflicts are governed by Academic code. According to Section 14.2, students with 3 or more finals in one day, or 4 or more finals in a 24 hour period, may negotiate to change the time of one of these finals. If you have time conflict in your final exam schedule and need to do Math 20480 final at another time, you must inform your instructor by **April 15**.

Honor Code: Examinations, homework, assignment and quizzes are conducted under the honor code. While collaboration in small groups in doing homework is permitted (and strongly encouraged) in this course, copying is

not. In particular, **copying from the Student Solutions Manual is a violation** of the Honor Code. Exams are closed book and are to be done completely by yourself with no help from others.

Homework & Assignment: Online Homework and assignment problems are assigned regularly. Their schedule is listed on Mobius or announced via email. Absolutely no late homework or assignment will be accepted. You are encouraged to work on these problems in groups, but all online homework and assignments must be turned in individually. Remember that you **will not learn anything by simply copying another student's work or the Student Solutions Manual**. The main purpose of homework and assignment is to help you learn the material and assess yourself. Experience shows that students who take their homework seriously do very well in the course because they have a better understanding of the material. For detailed homework and assignment instruction, please see attached information.

Class Attendance: A student who accumulates more than 3 unexcused absences may be given an F grade.

Classroom Policies: Please do your best to show up on time and quietly enter the room if you are late. Please remember to respect your peers who are here to learn. Indeed, class disruptions will **not** be tolerated and offending parties will be asked to leave. During lectures you are encouraged to actively participate by answering and asking questions.

MATH 20480 Course Work Policy

There are both online homework and paper-pencil assignments for this course.

Written Assignments are due in class according to the schedule posted on the Math 20480 website. The questions and problems to be turned in are posted on the course website. You are expected to submit your written assignment in the following manner:

- Your work has to be clearly and logically written; showing method of solution not just a final answer.
- Please staple your work together. It is your responsibility that your work stays stapled together securely.
- Any work falling short of the above expectations may not be graded.

Absolutely no late assignments will be accepted. If you need to attend a school related event, you may turn in your assignment early or arrange to have your peer turn it in on the day it is due. Exceptions are handled case by case.

Online Homework is assigned daily and is due at the end of the next class day. Their schedule posted on the Math 20480 homework website. The online system we are using is Mobius.

All online homework should be done using paper and pencil, and be treated the same manner as written assignments. We encourage you to keep a record of your work for material submitted online; these are helpful when you review for an exam. Absolutely no late homework will be accepted. Access the online homework at the Mobius through Canvas with your usual log-in: http://canvas.nd.edu/

Online Homework Submission Policies. All submission deadlines for online homework on Mobius are fixed. You are highly encouraged to SUBMIT your homework well ahead of deadlines. We DO NOT accept excuses like: "My computer/Webservers shut down just before I could submit my work on time." Save your answers as you enter them online. This ensures that no work is lost BEFORE the submission deadline. Enough "buffer" time is given to ensure timely submission of your work. All online homework are due at 12:00am (+2 hrs buffer) at the end of the next class day unless otherwise stated. In addition, after the deadline of a homework, you have 48 hours to complete a late homework to obtain up to 80% of the full score.

Homework Drop Three lowest (written+online) homework scores will be dropped.

Course Project: This a group project. Each group must turned in a typed write-up and give a short presentation of their findings.

Basic Algebra Rules

Exponential Rules:

$a^m \cdot a^n = a^{m+n}$ $\qquad$ $(ab)^m = a^m b^m$ $\qquad$ $\dfrac{a^m}{a^n} = a^{m-n}; \quad a \neq 0$

$a^0 = 1 \quad a \neq 0$ $\qquad$ $a^{1/m} = \sqrt[m]{a}$ $\qquad$ $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}; \quad b \neq 0$

$(a^m)^n = a^{mn}$

Distribution Law:

$a(b + c) = ab + ac$ $\qquad$ $\dfrac{a + b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$ $\qquad$ $\dfrac{a - b}{c} = \dfrac{a}{c} - \dfrac{b}{c}$

Quadratic Factoring:

$(a + b)^2 = a^2 + 2ab + b^2$ $\qquad$ $(a - b)^2 = a^2 - 2ab + b^2$

$a^2 - b^2 = (a - b)(a + b)$

Properties of Logarithm:

$\log_a(MN) = \log_a M + \log_a N$ $\qquad$ $\log_a\left(\dfrac{M}{N}\right) = \log_a M - \log_a N$ $\qquad$ $\log_a(M)^t = t \log_a M$

$\log_a a = 1$ $\qquad$ $\log_a 1 = 0$

$\log_a a^x = x$ $\qquad$ $a^{\log_a x} = x$

Change of Base: $\qquad \log_a M = \dfrac{\log_b M}{\log_b a}$

$\ln(MN) = \ln M + \ln N$ $\qquad$ $\ln\left(\dfrac{M}{N}\right) = \ln M - \ln N$ $\qquad$ $\ln(M)^t = t \ln M$

$\ln e = 1$ $\qquad$ $\ln 1 = 0$

$\ln e^x = x$ $\qquad$ $e^{\ln x} = x$

Summary of Differentiation Rules

The following is a list of differentiation formulae and statements that you have to know.

Product Rule:

$$(f(x)g(x))' = f(x)g'(x) + f'(x)g(x)$$

Quotient Rule:

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}$$

Chain Rule:

$$(f(g(x))' = f'(g(x))g'(x)$$

Derivative of Trigonometric Function:

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x \qquad \frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}$$

Derivative of Exponential and Logarithm Functions:

$$\frac{d}{dx}(e^x) = e^x \qquad \frac{d}{dx}(a^x) = a^x \ln(a) \qquad \frac{d}{dx}(\ln x) = \frac{1}{x}$$

Summary of Integration Rules

The following is a list of integral formulae and statements that you have to know.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C; \quad n \neq -1$$

$$\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C \qquad \int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int e^x\,dx = e^x + C \qquad \int e^{kx}\,dx = \frac{1}{k}e^{kx} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \sec x \tan x\,dx = \sec x + C$$

$$\int \csc x \cot x\,dx = -\csc x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C \qquad \int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

$$\int \frac{1}{1+x^2}\,dx = \arctan x + C \qquad \int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + C$$

Fundamental Theorem of Calculus

Let F an antiderivative of f i.e. $F'(x) = f(x)$. Then we have:

(1) $\displaystyle\int_a^b f(x)\,dx = F(b) - F(a) = [F(x)]_a^b$

(2) Total change in F when x changes from a to $b = F(b) - F(a) = \displaystyle\int_a^b F'(x)\,dx$

Integration by Parts:

(1) $\displaystyle\int u\,dv = uv - \int v\,du$ (2) $\displaystyle\int_a^b u\,dv = [uv]_a^b - \int_a^b v\,du$

Math 20480 Syllabus

Introduction to Dynamical Systems for Scientist

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1. Matrix Algebra
   Solving a System of Linear Equations – Gaussian Elimination
   Inverse of a Square Matrix
   Linear Transformations
   Vector Space, Subspaces and Bases

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2. Eigenvalue & Eigenvectors
   Canonical Forms of a Square Matrix – Diagonal, Jordan and Normal matrix
   Powers of a n x n Matrix – Application of Canonical Forms
   Solving Homogeneous Linear Systems of Difference Equations with n variables

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3. Solving Homogeneous Linear n-th Order Difference Equations
   Application to Leslie Model and Lefkovitch Model (Matrix Population Dynamics)
   Application to WAIFW matrix (cross infection Model and transmission rates)

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4. Mathematical Models with Linear Systems of Differential Equations
   - Richardson's Theory of War
   - Predator and Prey model
   - Competition model
   - Spring-mass model
   - Ackerman Glucose Metabolism model
   - Compartmental Infectious Disease model

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5. Exponentiation of a Square Matrix
   Solving Linear Systems of Differential Equations in n variables
   Solving Linear nth Order Differential Equations in one variable

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6. Qualitative Analysis of the solutions for a Linear Systems of Differential Equations with 2 variables
   - Equilibrium Points
   - Phase Portraits
   - Classification of Equilibrium Points (Stable/Unstable)
   - Application to Richardson's Theory of War

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7. Non-linear Systems of Differential Equations with 2 variables
   - Linearization at Equilibrium Points
   - Stability of Equilibrium Points
   - Application to Mathematical Models

Access for Calculus B online Homework system Mobius Assessments

The online homework system Mobius is accessed through Canvas. Follow the steps below to logged into Mobius

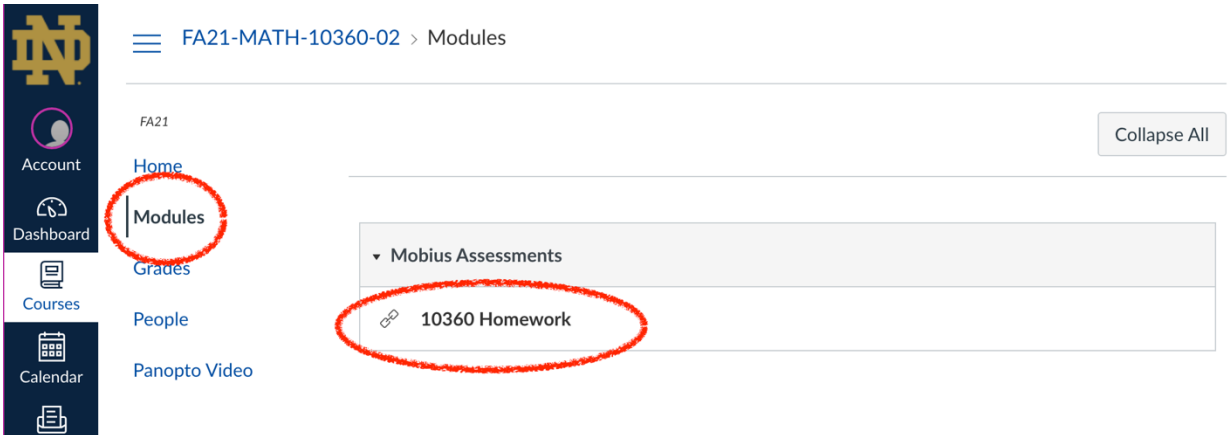
(a) Access Canvas through InsideND by searching for the Canvas App or using the link:

https://canvas.nd.edu/

(b) Log into Canvas using your NetID and password.

(c) Find your Canvas Calc B section using the navigation bar on the left.

(d) In your Calc B section, click on "Module". You will see the link to 10360 Homework in the Mobius Assessment module. See figure below.



(e) Click on the "10360 Homework" link and launch Mobius.

(f) Mobius will launch in a new tap.

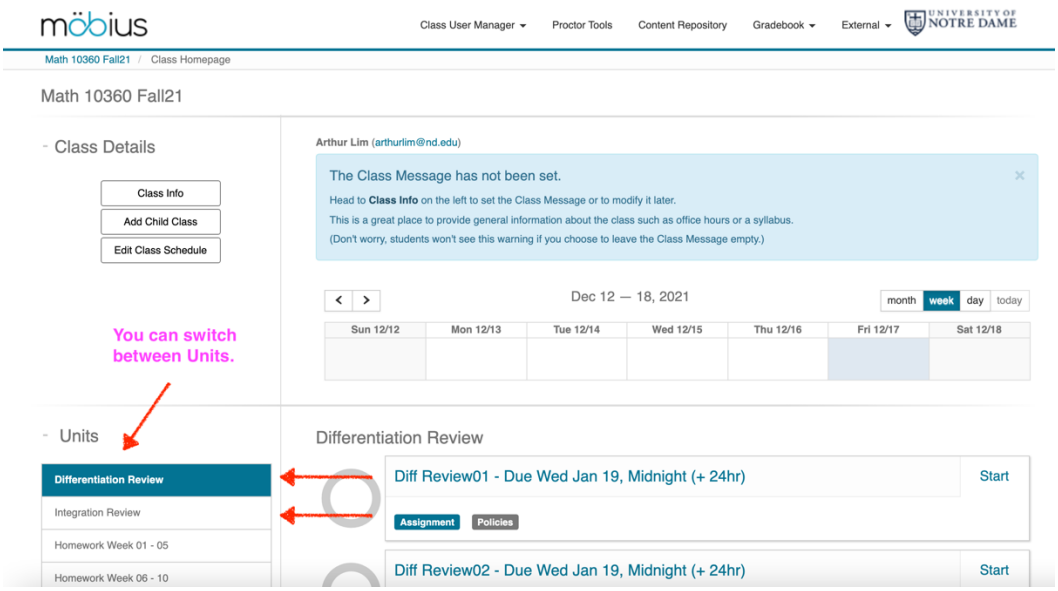
(g) There are three units of homework available. You can toggle between the units to see the sets of homework available. See figure below.

Differentiation Review – Three sets and 25 points each.

Integration Review – Two sets and 25 points each.

Gateway Prep – Three sets and 25 points each.

You have unlimited attempts till all questions are correct. Correct answers of previous attempts are saved for exercises with no time limit.



The due dates are as stated on title of each homework set. The first due date is Wednesday midnight + 24 hour grace time. This means the work should be done by midnight Thursday and the homework will close sometime after midnight Tuesday Jan 20. We are using the due date to pace your learning so please schedule time to keep the due dates.

After you submitted a set of homework, you can see details of the graded work by clicking on the score or see details in the gradebook in Mobius. See an illustrative figure below. Explore the links.

Learn from your mistakes in the provided solution. Understand the rules applied and process of solution but DO NOT just follow a “cook book” process or mere pattern recognition. Every step of the solution has a reason.