Math 20480 – Example Set 01A

1. The chance of a person moving from City **A** to City **B** is 20% and from City **B** to City **A** is 10% by the end of each year. Let $x(n)$ and $y(n)$ denote the population (in thousands) of City **A** and City **B** respectively after n years from now. If the current population of City **A** is 10 thousand and that of City **B** is 12 thousand, write down a system of equations that describes the population of City **A** and City **B** if annual migration rates persists.

2. Answer the following questions for the given matrices:

$$A = \begin{pmatrix} 1 & -2 \\ -7 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 3 & -5 \\ -3 & 1 & 4 \\ 5 & -4 & 6 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad E = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} \qquad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(a). Perform the multiplications whenever it makes sense: (i) AB, (ii) BC, (iii) CI_3, (iv) DE and, (v) BED.

(b). How are D and E related?

(c). Find the transpose C^T of matrix C.

(d). Evaluate the expressions $3A + 2B$ and $(3A + 2B)B$.

Basic Matrix Operations and their Properties.

(Scalar Multiplication). Let s be a scalar (number). Let A and B be matrices of the same size. The scalar multiplication of s and A, sA is done entry-wise. Moreover we have the following distribution properties: If t and r are scalars then

(S1). $t(sA) = (ts)A = tsA$

(S2). Distributive Law: $r(sA + tB) = rsA + rtB$

(Matrix Multiplication). Let $A = (a_{ij})$ be a $k \times n$ matrix, $B = (b_{ij})$ be a $n \times m$ matrix. Then the product M of the matrices A and B is a $k \times m$ matrix with ij-entry m_{ij} given by

$$m_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{in}b_{nj}$$

for all $1 \leq i \leq k$ and $1 \leq j \leq m$.

(M1). Transitive Law: Let A be a $k \times n$ matrix, B be a $n \times m$ matrix, and C be a $m \times p$ matrix, then the product $A(BC) = ABC = (AB)C$ is a $k \times p$ matrix.

(M2). Distributive Law: Let A be a $k \times n$ matrix, B and C be $n \times m$ matrices, then $A(B+C) = AB+AC$ is a $k \times m$ matrix.

Math 20480 – Example Set 01B

1. (Age-structured Population model - The Leslie Model) The female three-spine stickleback fish usually live up to four years. It is estimated that the survival rate of a female stickle back in their first, second, and third years is 5%, 10%, 15%. It also is estimated that each year a one-year old female produces 400 female offsprings, a two-year old female 450 female offsprings, a three-year old female 500 female offsprings, and a four-year old female 550 female offsprings. Construct a population model with the given survive rates and fertility rates (fecundity) of a female three-spine stickleback given below. Draw a life-cycle graph to describe your model and write down your equations in matrix form. Discuss what must be done to solve such a system of equations.

2. Use Gaussian elimination to find all possible solutions to the following system of linear equations.

a.

$$\begin{aligned} x + y + 2z &= 8 \\ x - 2y - z &= -1 \\ x - y + z &= 4 \end{aligned}$$

b.

$$\begin{aligned} 2x + 3y - 4z &= -4 \\ 3x - 5y + z &= 20 \\ x + 11y - 9z &= -28 \end{aligned}$$

c.

$$\begin{aligned} x + 9y - 11z &= 0 \\ 2x + 4y - 3z &= -4 \\ 3x - y + 5z &= 4 \end{aligned}$$

d.

$$\begin{aligned} x + 9y - 11z &= 0 \\ 2x + 4y - 3z &= -4 \end{aligned}$$

Math 20480 – Matrices Example

2. Answer the following questions for the given matrices:

$$A = \begin{pmatrix} 1 & -2 \\ -7 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 3 & -5 \\ -3 & 1 & 4 \\ 5 & -4 & 6 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad E = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix} \qquad I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(a). Perform the multiplications whenever it makes sense:

(i) AB, (ii) BC, (iii) CI_3, (iv) DE and (v) BED.

(b). How are D and E related?

(c). Find the transpose C^T of matrix C.

(d). Evaluate the expressions $3A + 2B$ and $(3A + 2B)B$.

Math 20480 – Example Set 01C

1. (Stage-structured population model - Lefkovitch model) The janitor fish, a kind of armored catfish (Hypostomus plecostomus), is native to tropical Central America and South America. They mostly live in fresh water or brackish water rivers. Priced in the aquarium trade, the janitor fish is commonly seen in various Asian countries like Singapore, the Philippines and Hong Kong. A very recent studies in the Philippines made the following observations. The ratio of male vs female is 1:1. Annual ecundity averages at 2216 eggs per female but only half will hatch due to limited male brooding capability. Moreover only 47% of the hatchlings (new frys) are viable. The monthly survival rates for a fry (1-2 month duration) is 62% and 40.3% graduates to a juvenile (2 yrs duration), for a juvenile 99.18% and only 0.62% graduates to an adult fish (up to 15 yrs in captivity), and for an adult fish 98.9%. [1]

1a. Draw the lifecycle graph for janitor fish filling in the monthly fecundity and survival rates. Assume that census are taken after eggs are laid.

1b. Write down the system of linear equations and the associated Lefkovitch matrix

2. Use Gaussian elimination to find all possible solutions to the following system of linear equations.

a.

$$\begin{aligned} x + y + 2z &= 8 \\ x - 2y - z &= -1 \\ x - y + z &= 4 \end{aligned}$$

b.

$$\begin{aligned} 2x + 3y - 4z &= -4 \\ 3x - 5y + z &= 20 \\ x + 11y - 9z &= -28 \end{aligned}$$

c.

$$\begin{aligned} x + 9y - 11z &= 0 \\ 2x + 4y - 3z &= -4 \\ 3x - y + 5z &= 4 \end{aligned}$$

[1] Reference: Vallejo, Jr; Soriano; "A matrix population model of the "janitor fish" Pterygoplichthys (Pisces: Loricariidae) in the Marikina River, Luzon island, Philippines and the possibility of controlling this invasive species"; Philippine Science Letters; vol. 4, no. 1, 2011