Math 20480 – Example Set 04A

(Eigenvalues and Eigenvectors). Let A be a $n \times n$ square matrix. A non-zero n-th tuple $\vec{u}$ is called an eigenvector of A corresponding to eigenvalue λ if $A\vec{u} = \lambda\vec{u}$.

(Characteristic Polynomial). Let A be a $n \times n$ square matrix. The polynomial $|A - \lambda I_n|$ is called the characteristic polynomial of A. The zeros of the characteristic polynomial that is, the solutions of

$$|A - \lambda I_n| = 0$$

are the eigenvalues of A.

1a. Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & -2 \\ -1 & 0 \end{pmatrix}$$

1b. Find an eigenvector corresponding to each of the above eigenvalues.

1c. Find a 2×2 matrix P such that

$$A = P \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} P^{-1}$$

1d. Solve the system of difference equations

$$\begin{aligned} x(n) &= x(n-1) - 2\,y(n-1) \qquad & x(0) &= 1 \\ y(n) &= -x(n-1) & y(0) &= -8 \end{aligned}$$

Math 20480 – Example Set 04B

1. The chance of a person moving from City **A** to City **B** is 20% and from City **B** to City **A** is 10% by the end of each year. Let $x(n)$ and $y(n)$ denote the population (in thousands) of City **A** and City **B** respectively after n years from now.

1a. If the current population of City **A** is 10 thousand and that of City **B** is 12 thousand, write down a system of equations that describes the population of City **A** and City **B** if annual migration rates persists.

1b. Find a formula for the size of the cities for any year n.

1c. What could you say about the cities as time progresses?

Math 20480 – Example Set 04C

1. Consider the matrix $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$.

1a. Find characteristic polynomial of A.

1b. Find the eigenvectors of A.

1c. Diagonalize the matrix A. That is find a 3×3 matrix P such that $A = PDP^{-1}$ where D is a 3×3 diagonal matrix.

1d. Find the general solution of the system of difference equations below.

$$\begin{aligned} x(n) &= 4\,x(n-1) + y(n-1) - z(n-1) \\ y(n) &= 2\,x(n-1) + 5\,y(n-1) - 2\,z(n-1) \\ z(n) &= x(n-1) + y(n-1) + 2\,z(n-1) \end{aligned}$$

1e. Find the solution of the system of difference equations below.

$$\begin{aligned} x(n) &= 4\,x(n-1) + y(n-1) - z(n-1) \qquad & x(0) &= 1 \\ y(n) &= 2\,x(n-1) + 5\,y(n-1) - 2\,z(n-1) \qquad & y(0) &= 0 \\ z(n) &= x(n-1) + y(n-1) + 2\,z(n-1) \qquad & z(0) &= -1 \end{aligned}$$