**Math 20480 – Example Set 05A**

**1.** Consider the matrix $A = \begin{pmatrix} 3 & 0 & 0 \\ 1 & 4 & 4 \\ 0 & 0 & 5 \end{pmatrix}$.

Find the following

**1a.** The characteristic polynomial.

**1b.** The eigenvectors.

**1c.** Diagonalize $A$.

# Math 20480 – Example Set 05B

**(Jordan Matrix).** We call any matrix in the form $\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$ a Jordan matrix. It could be thought of as an "almost diagonal" matrix

**(Repeated Real Eigenvalues).** In the case when a $2 \times 2$ matrix $A$ has one eigenvalue $\lambda$ and only one representative for the eigenvector can be found, then $A$ cannot be diagonalized. When this happens we associate $A$ to a Jordan matrix $J = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$ such that $A = PJP^{-1}$.

Here $P = (\vec{u}|\vec{v})$ where $\vec{u}$ is an eigenvector of $A$ and $\vec{v}$ is such that $(A - \lambda I)\vec{v} = \vec{u}$. We say that $J$ is the Jordan normal form of $A$.

We note that $(A - \lambda I)^2\vec{v} = (A - \lambda I)\vec{u} = 0$. Because of this, we call such $\vec{v}$ a generalized eigenvector of $A$ corresponding to $\lambda$.

**1a.** Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 5 & -1 \\ 4 & 1 \end{pmatrix}$$

**1b.** Find an eigenvector corresponding to each of the above eigenvalues.

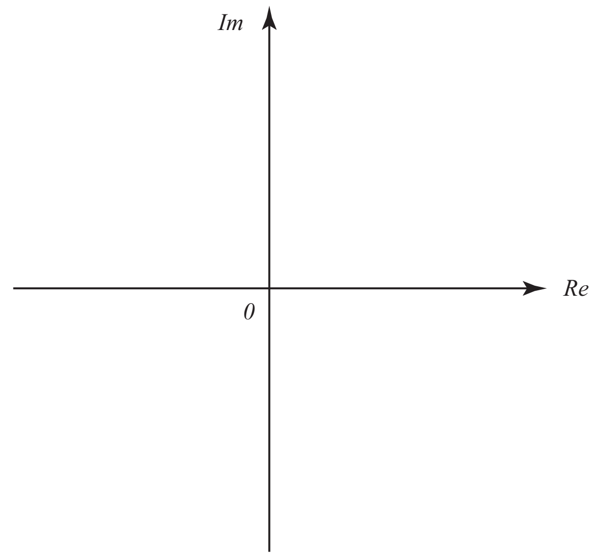
**1c.** Find a $2 \times 2$ matrix $P$ such that

$$A = P \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix} P^{-1}$$

**1d.** Solve the system of difference equations

$$\begin{aligned} x(n) &= 5x(n-1) - y(n-1) & \qquad x(0) &= 1 \\ y(n) &= 4x(n-1) + y(n-1) & \qquad y(0) &= -2 \end{aligned}$$

# Math 20480 – Example Set 05C




**(Complex Number).** A number of the form $z = a + bi$ where $a$ and $b$ are real numbers and $i = \sqrt{-1}$ is called a complex number. The complex number $z = a + bi$ can be represented as a point on the complex plane (or Argand plane) with coordinates $(a, b)$.

**(Modulus of a Complex Number).** The length or modulus $|z|$ of the complex number $z = a + bi$ is given by the distance of the point $(a, b)$ from the origin in the complex plane. So we have the formula

$$|z| = \sqrt{a^2 + b^2}$$

**(Polar Form of a Complex Number).** Consider the complex number $z = a + bi$. Its coordinates $(a, b)$ in the complex plane can be written in polar coordinates $(|z|\cos\theta, |z|\sin\theta)$ where $0 \leq \theta \leq 2\pi$ such that $\tan\theta = \dfrac{b}{a}$. Therefore we may write

$$z = |z|(\cos\theta + i\sin\theta)$$

Using the Maclaurin series of $\cos x$, $\sin x$, and $e^x$, we may show that

$$e^{i\theta} = \cos\theta + i\sin\theta \quad \text{(Euler's Formula)}$$

Therefore we have

$$z = a + bi = |z|e^{i\theta}$$

where $|z| = \sqrt{a^2 + b^2}$ and $0 \leq \theta \leq 2\pi$ such that $\tan\theta = \dfrac{b}{a}$.

**(Conjugate of a Complex Number).** The conjugate $\bar{z}$ of the complex number $z = a + bi$ is defined as $\bar{z} = a - bi$. Note that $|\bar{z}| = |z|$.

**1.** For the complex numbers below (i) draw them in the complex plane, (ii) find their modulus, (iii) give their polar form, and (iv) find their conjugates.

**(a)** $-3i$

**(b)** $-1 + i$

**(c)** $-1 - \sqrt{3}i$

**(d)** $e^{i\frac{\pi}{6}}$