Math 20480 – Example Set 06A

Complex Eigenvalues). In the case when a 2×2 matrix A has eigenvalues $\alpha \pm \beta\, i$ with eigenvectors $\vec{p} \pm \vec{q}\, i$, then A be diagonalized but with complex numbers as entries in PDP^{-1} in the usual way.

This is not very helpful in our case as we are working with real values only. When this happens we associate A to a real-entry normal matrix $S = \begin{pmatrix} \alpha & \beta \\ -\beta & \alpha \end{pmatrix}$ such that $A = PSP^{-1}$.

Here $P = (\vec{p}|\vec{q})$ where $\vec{p} + \vec{q}\, i$ is an eigenvector of A corresponding to $\alpha \pm \beta\, i$.

1a. Find the eigenvalues of the matrix

$$A = \begin{pmatrix} -7 & -6 \\ 15 & 11 \end{pmatrix}$$

1b. Find an eigenvector corresponding to each of the above eigenvalues.

1c. Find a 2×2 matrix P such that

$$A = P \begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix} P^{-1}$$

Math 20480 – Example Set 06B

1a. Write the matrix $S = \begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix}$ in the form $\begin{pmatrix} r\cos\theta & r\sin\theta \\ -r\sin\theta & r\cos\theta \end{pmatrix}$.

1b. Solve the system of difference equations

$$\begin{aligned} x(n) &= -7x(n-1) - 6y(n-1) \qquad & x(0) &= -1 \\ y(n) &= 15x(n-1) + 11y(n-1) \qquad & y(0) &= 1 \end{aligned}$$

2a. Consider the matrix

$$B = \begin{pmatrix} -3 & -1 & 1 \\ 2 & 1 & -2 \\ 2 & 4 & -4 \end{pmatrix}$$

Find the invertible matrix P such that $B = PCP^{-1}$ where C is a canonical form of B.

2b. Solve the system of difference equations

$$\begin{aligned} x(n) &= -3x(n-1) - y(n-1) + z(n-1) \\ y(n) &= 2x(n-1) + y(n-1) - 2z(n-1) \\ z(n) &= 2x(n-1) + 4y(n-1) - 4z(n-1) \end{aligned}$$