Math 20480 – Example Set 07A

Jordan Form for Higher Dimensions

1. Consider the matrices

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 1 & 3 & 4 \\ 0 & 0 & 5 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

For each of the matrices above, find the following

(a) The characteristic polynomial.

(b) The eigenvectors and generalized eigenvectors if any.

(c) Write the matrix in its canonical forms.

2. Find the general solution of the system of difference equations below.

$$\begin{aligned} x(n) &= 3\,x(n-1) \\ y(n) &= x(n-1) + 3\,y(n-1) + 4\,z(n-1) \\ z(n) &= 5\,z(n-1) \end{aligned}$$

3. Find the particular solution of the system of difference equations below.

$$\begin{aligned} x(n) &= 2\,x(n-1) + z(n-1); & x(0) &= 1 \\ y(n) &= x(n-1) + 2\,y(n-1) + z(n-1); & y(0) &= 2 \\ z(n) &= 2\,z(n-1); & z(0) &= 3 \end{aligned}$$

Math 20480 – Example Set 07B

1.

$$D = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 1 & 5 & -1 & -6 \\ 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{pmatrix}; \quad E = \begin{pmatrix} 2 & 0 & 0 & 1 \\ 1 & 3 & -1 & -2 \\ 1 & 1 & 1 & -2 \\ 0 & 0 & 0 & 2 \end{pmatrix}; \quad F = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 1 & 5 & -1 & -6 \\ 1 & 1 & 1 & -2 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

For each matrices above, find the following:

(a) Find characteristic polynomial and the eigenvalues of the matrix.

(b) Find the eigenvectors and generalized eigenvectors if any.

(c) Find the canonical form J associated to the matrix and a matrix P such that the given matrix can be written in the form PJP^{-1}.

Powers of Larger Jordan Matrices

$$\begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}^n = \begin{pmatrix} \lambda^n & n\lambda^{n-1} & \binom{n}{2}\lambda^{n-2} \\ 0 & \lambda^n & n\lambda^{n-1} \\ 0 & 0 & \lambda^n \end{pmatrix} \qquad \text{Here } \binom{n}{2} = \frac{n!}{2!(n-2)!} = \frac{n(n-1)}{2}$$

$$\begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix}^n = \begin{pmatrix} \lambda^n & n\lambda^{n-1} & \binom{n}{2}\lambda^{n-2} & \binom{n}{3}\lambda^{n-3} \\ 0 & \lambda^n & n\lambda^{n-1} & \binom{n}{2}\lambda^{n-2} \\ 0 & 0 & \lambda^n & n\lambda^{n-1} \\ 0 & 0 & 0 & \lambda^n \end{pmatrix} \qquad \text{Here } \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Math 20480 – Example Set 07C

1.

$$G = \begin{pmatrix} 2 & 1 & -1 & -2 \\ -1 & -1 & 1 & 7 \\ -1 & 0 & 2 & 1 \\ 0 & -1 & 0 & 4 \end{pmatrix}$$

(a) Find characteristic polynomial and the eigenvalues of the matrix G.

(b) Find the eigenvectors and generalized eigenvectors if any.

(c) Find the canonical form J associated to G and a matrix P such that $G = PJP^{-1}$.

2. Find the general solution of the system of difference equations below.

$$\begin{array}{rrrrrcrr}
x(n) = & 2\,x(n-1) & +\ y(n-1) & -\ z(n-1) & -\ 2\,w(n-1) & \quad & x(0) = & 1 \\
y(n) = & -\,x(n-1) & -\ y(n-1) & +\ z(n-1) & +\ 7\,w(n-1) & & y(0) = & 0 \\
z(n) = & -\,x(n-1) & & +\ 2\,z(n-1) & +\ w(n-1) & & z(0) = & -1 \\
w(n) = & & -\ y(n-1) & & +\ 4\,w(n-1) & & w(0) = & 1
\end{array}$$