

**Matrix Exponential Formulas (Real Eigenvalue Case).**

$$\exp\begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix} = \begin{pmatrix} e^\lambda & 0 \\ 0 & e^\mu \end{pmatrix}; \quad \exp\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} e^\lambda & e^\lambda \\ 0 & e^\lambda \end{pmatrix}$$

$$\exp\left[\begin{pmatrix} \lambda & 0 \\ 0 & \mu \end{pmatrix} \cdot t\right] = \exp\begin{pmatrix} \lambda t & 0 \\ 0 & \mu t \end{pmatrix} = \begin{pmatrix} e^{\lambda t} & 0 \\ 0 & e^{\mu t} \end{pmatrix};$$

$$\exp\left[\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \cdot t\right] = \exp\begin{pmatrix} \lambda t & t \\ 0 & \lambda t \end{pmatrix} = \begin{pmatrix} e^{\lambda t} & te^{\lambda t} \\ 0 & e^{\lambda t} \end{pmatrix}$$

$$\exp\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} e^{\lambda_1} & 0 & 0 \\ 0 & e^{\lambda_2} & 0 \\ 0 & 0 & e^{\lambda_3} \end{pmatrix}; \quad \exp\begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix} = \begin{pmatrix} e^\lambda & e^\lambda & e^\lambda \\ 0 & e^\lambda & e^\lambda \\ 0 & 0 & e^\lambda \end{pmatrix}$$

$$\exp\left[\begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \cdot t\right] = \exp\begin{pmatrix} \lambda_1 t & 0 & 0 \\ 0 & \lambda_2 t & 0 \\ 0 & 0 & \lambda_3 t \end{pmatrix} = \begin{pmatrix} e^{\lambda_1 t} & 0 & 0 \\ 0 & e^{\lambda_2 t} & 0 \\ 0 & 0 & e^{\lambda_3 t} \end{pmatrix}$$

$$\exp\left[\begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix} \cdot t\right] = \exp\begin{pmatrix} \lambda t & t & 0 \\ 0 & \lambda t & t \\ 0 & 0 & \lambda t \end{pmatrix} = \begin{pmatrix} e^{\lambda t} & te^{\lambda t} & t^2 e^{\lambda t} \\ 0 & e^{\lambda t} & te^{\lambda t} \\ 0 & 0 & e^{\lambda t} \end{pmatrix}$$

$$\exp\left[\begin{pmatrix} \lambda & 1 & 0 & 0 \\ 0 & \lambda & 1 & 0 \\ 0 & 0 & \lambda & 1 \\ 0 & 0 & 0 & \lambda \end{pmatrix} \cdot t\right] = \exp\begin{pmatrix} \lambda t & t & 0 \\ 0 & \lambda t & t \\ 0 & 0 & \lambda t \end{pmatrix} = \begin{pmatrix} e^{\lambda t} & te^{\lambda t} & t^2 e^{\lambda t} \\ 0 & e^{\lambda t} & te^{\lambda t} \\ 0 & 0 & e^{\lambda t} \end{pmatrix}$$

1. (cf. Q1d/05B) Find the solution of the system of differential equations:

$$\begin{aligned} x'(t) &= 5x(t) - y(t); & x(0) &= 1 \\ y'(t) &= 4x(t) + y(t); & y(0) &= -2 \end{aligned}$$

2. (cf. Q2/07A) Solve the initial value problem:

$$\begin{aligned} x' &= 3x \\ y' &= x + 3y + 4z \\ z' &= 5z \end{aligned}$$