Math 102 Homework #4 Oct. 1, 1993

1. Suppose that you have an unlimited quantity of 14 lb and 28 lb weights. Using a pan balance find

   (a) the smallest weight of a bag of sugar that can be weighed.

   (b) Show that a 63 lb bag of sugar can be weighed.

2. (a) A booth in a mall sells cupcakes for 17 cents each. A neighboring booth sells donuts for 13 cents each. The owner of the cupcake booth craves some donuts but has no money to buy them. The owner then proposes to trade cupcakes for donuts. What is the smallest number of cupcakes involved in the transaction if neither owner loses any money on the deal?

   (b) The owner of the cupcake booth finds a penny lying on the ground. Since nobody claims it, he uses it in the transaction described above. What is the smallest number of cupcakes involved in this new transaction?

3. (a) By consulting your table of squares and primes form a conjecture about the last (units) digit of a number if it is to be a perfect square.

   (b) Use your conjecture (assumed to be true) to show that the number 31176899843573 is not a perfect square.

   (c) Does your conjecture enable you to tell when a number is a perfect square?

4. Examine your table of squares and form a conjecture concerning squares of even numbers and squares of odd numbers.

5. Divide some of the odd squares by four and record the remainder in each case. Form a conjecture concerning the remainder when an odd square is divided by four and test this conjecture with the remaining squares.

6. (a) Consulting the table of primes form a conjecture about the last digit in a number in order that it be a prime number, and use the conjecture (assumed to be true) to show that the number 911784352996 cannot be a prime.

   (b) Does your conjecture enable you to tell where a number is a prime? (Hint: check the table of squares for a counter example.)