Math 102 Homework #5 Due Oct. 14, 1996

1. Consulting your tables of squares and primes find all primes which are sums of two squares.

2(a). Find all primes of the form $4N + 1$ and all those of the form $4N + 3$.

(b) Comparing these results with the results of exercise 1, formulate a conjecture concerning the form of all odd primes which are sums of squares.

3(a). Consulting the table of squares we see that if we set $a = 3, b = 4, c = 5$ then $a^2 + b^2 = c^2$. It is also true if we double or triple the values of a, b and c that $a^2 + b^2 = c^2$.

Formulate a theorem by which infinitely many triples a, b, c can be found from any given triple.

(b) Interpret the results of part (a) geometrically.

(c) Using the table find another triple, other than the triples found in part (a) from the triple 3, 4, 5. Interpret this triple geometrically. (All triples a, b, c such that $a^2 + b^2 = c^2$ are called "Pythagorean Triples".)

4. Consulting the table of squares we see that there are pairs of numbers there which add up to squares. For instance $64 + 36 = 100 = 10^2$, $144 + 256 = 400 = 20^2$. However none of these pairs consist of two odd numbers. Show that there are no pairs of odd squares which add up to a square. (See problem 3 above)

5. (a) What are the possible remainders when a natural number, or zero, is divided by 6?

(b) We have seen that the remainders when zero or a natural number is divided by 2 are 0 and 1. This result was used to classify numbers by $N = 2n + 0$ (even) and $N = 2n + 1$ (odd). Using the result of part (a) above classify numbers, in a similar way, according to their remainders when divided by 6.

6. Examination of the table of primes shows that for all primes greater than 3 in the table the remainder upon division by 6 is either 1 or 5. Using the result of (b) above show that this is true for <u>every</u> prime greater than 3.

7. Suppose that a, b and c are natural numbers and that a divides the product bc of b and c. Does it follow that a must divide either b or c? If not give a counter example.

8. Suppose that the number a in the preceding problem is a prime number. Does it now follow that a divides b or c?