UNIVERSITY OF NOTRE DAME
DEPARTMENT OF MATHEMATICS

Friday, March 23, 2001
Math. 103 - Processes of Mathematical Thought
Exam 2 - Prof. Borelli

NOTE: This is an OPEN BOOK exam, **but** pocket calculators are not allowed, nor is collaboration among students. **You are under the University's Honor Code.**

• •

I. ***(20 pts.)*** For each of the four configurations shown below of the "roadtoy" game studied in class give the number of "pivots" needed to reach the desired alphabetical order. You may assume that the tiles not shown are already in alphabetical order. **Be sure to**:

(a) if you use just one pivot, identify it by showing the four tiles being pivoted.

(b) if you use a sequence of 17 pivots to change the parity, identify the first one as explained in **(a)**, and clearly identify the relevant result.

(c) if you use a sequence of 8 pivots to "rotate" three consecutive tiles, identify the direction of rotation and which tile is the zero-th tile.

1. *(5 pts.)*

.....	L	M	N	O	P	Q	T	S	R	A	B	.....

2. *(5 pts.)*

.....	L	M	N	O	P	T	Q	S	R	A	B	.....

3. *(5 pts.)*

.....	L	M	N	O	R	P	Q	S	T	A	B	.....

4. *(5 pts.)*

.....	L	M	N	O	R	P	S	Q	T	A	B	.....

II. ***(20 pts.)*** For each of the two configurations shown below of the "roadtoy" game studied in class a number of "pivots" is shown which can be used to reach the desired alphabetical order. Identify precisely which pivots to use so that the desired alphabetical order is achieved with the number of pivots shown. You may assume that the tiles not shown are already in alphabetical order. **Be sure to** identify sequences of pivots according to rules **(a), (b)** and **(c)** stated in question **I.**

1. *(10 pts.)*

·····	L	M	N	P	Q	O	T	R	S	A	B	·····

Answer: 16 pivots

2. *(10 pts.)*

·····	L	M	N	O	P	T	Q	R	S	A	B	·····

Answer: 16 pivots

III. ***(20 pts.)*** Consider the permutation on 8 letters shown below:

$$P = (HEGADCB)(GABCDFE)(DEABFGH)(HFDBACEG)$$

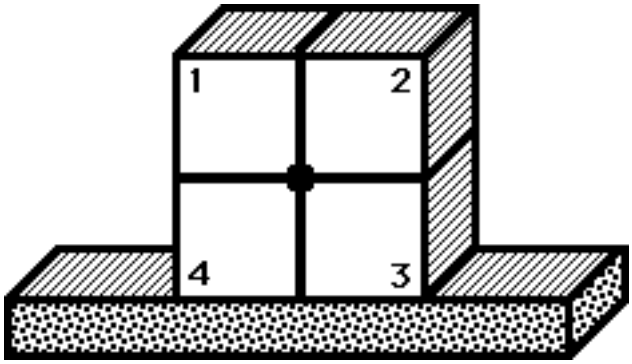
1. *(5 pts.)* What is the smallest number of transpositions needed to represent **P**?

2. *(5 pts.)* Represent **P** as the product of seven transpositions

3, *(5 pts.)* Represent **P** as the product of eight transpositions.

4. *(5 pts.)* What is the largest number of transpositions one can use to represent **P**?

IV. ***(20 pts.)*** Consider the toy shown below, which consists of a "box" sitting on a pedestal, with four tiles on which the letters **A, H, M, T** appear, initially at random.



You are allowed to interchange any two adjacent tiles, but **not** two diagonally opposite tiles.

A. *(10 pts.)* Show that any two non-adjacent tiles can also be interchanged using the four 'adjacent tile' interchanges. (*Hint:* for example, (12)(14)(12) = ...)

B. *(10 pts.)* Now show that the toy allows you to achieve the configuration

M	**A**
T	**H**

regardless of the starting configuration. (*Hint:* a statement shown in class about permutations and transpositions may help.)

V. ***(20 pts.)*** Messages consisting of the 26 letters of the alphabet (all in uppercase) and one blank space are being coded using the permutation **YME&~U** shown below (note that the cycles are NOT disjoint. The symbol **ß** stands for the one blank space.)

YME&~U = (THEßQUICK)(BROWN)(FOX)(JUMPSßOVER)(THEßLAZYDOG)

1. *(8 pts.)* Using **YME&~U** code the message **"IO SONO XXXX"** (Here XXXX stands for the first four letters of your first name. If three or less, use extra blanks.)

2. *(12 pts.)* The message shown below has been coded using **YME&~U**. Decode it.

JGBQEWXBZEC