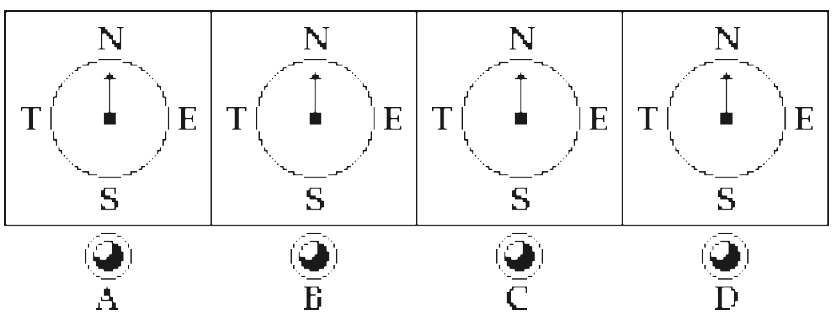
UNIVERSITY OF NOTRE DAME
DEPARTMENT OF MATHEMATICS

MATH. 103 - PROCESSES OF MATHEMATICAL THOUGHT
FINAL EXAMINATION *May 8, 2001*

NOTE: This is an **OPEN BOOK** exam, **but** you **MAY NOT** use the knowledge nor the work of your fellow students. **You are under the University's Honor Code.**
Be sure to SHOW ALL YOUR WORK!

I. **(25 pts.)** A piece of machinery consists of the four dials shown below, acted upon by the four push-buttons A, B, C and D also shown. When acted upon the hand of a dial moves clockwise 90 degrees. Pushbutton A controls the first two (from the left) dials, B controls the first three (also from the left.) The control of buttons D and C is in symmetry with that of A and B respectively, that is, D acts like A, but on the right, and C acts like B, but on the right. When the machine is at rest all four hands point to the letter N. You are to find which buttons need to be pushed how many times in order for the machine to spell **SENT** (i.e. first hand on left points to S, second points to E, etc.) without unnecessary repetitions of buttons.

A. **(6 pts.)** Set up the system whose solution will help you accomplish your task. Make sure you identify the modulus**.**
B. **(12 pts.)** Solve the system.using the Gauss-Jordan elimination method.
C. **(7 pts.)** Perform the required task



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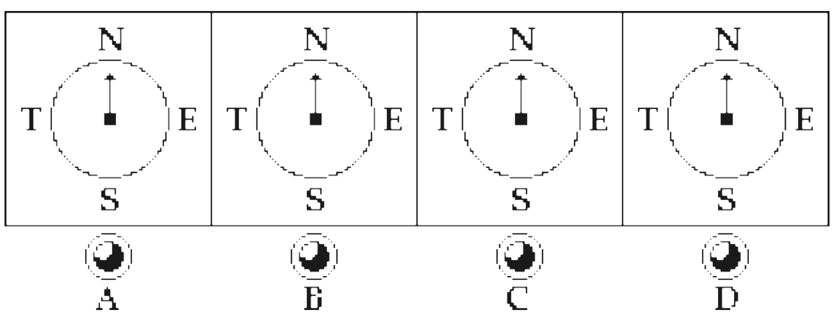
A. **(6 pts.)** Set up the system whose solution will help you accomplish your task. Make sure you identify the modulus**.**

B. **(12 pts.)** Solve the system.using the Gauss-Jordan elimination method.

C. **(7 pts.)** Perform the required task



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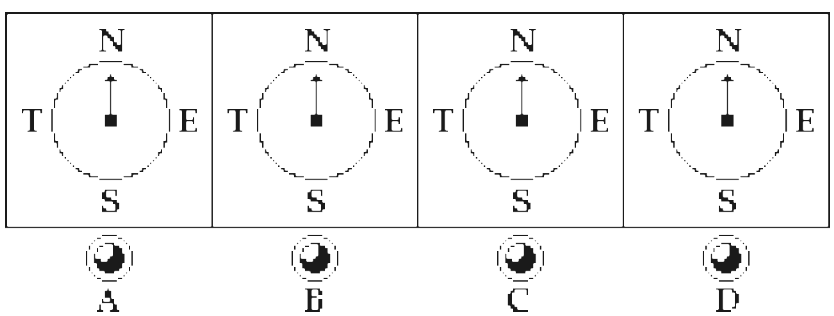
A. **(6 pts.)** Set up the system whose solution will help you accomplish your task. Make sure you identify the modulus**.**

B. **(12 pts.)** Solve the system.using the Gauss-Jordan elimination method.

C. **(7 pts.)** Perform the required task



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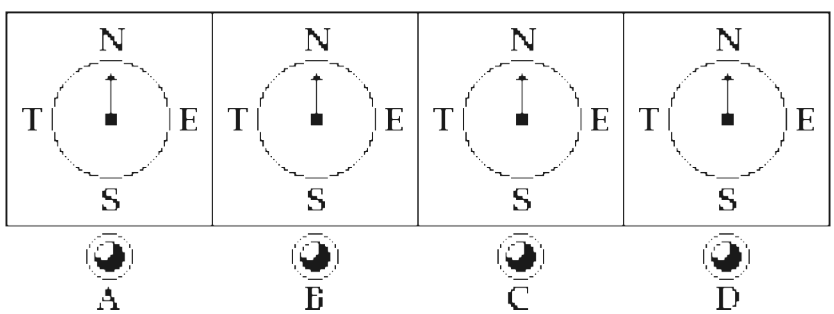
A. **(6 pts.)** Set up the system whose solution will help you accomplish your task. Make sure you identify the modulus**.**

B. **(12 pts.)** Solve the system.using the Gauss-Jordan elimination method.

C. **(7 pts.)** Perform the required task



II. **(15 pts.**) This question refers to the "original" version of the nine-button game, the version we studied first in class. Each button can be either ON or OFF. Shown below is a **"target"** configuration which was just reached, and you are told that all nine buttons had to be pushed, each one exactly once, to achieve the target shown. Give the starting configuration. Explain your work (in other words, just showing the starting configuration is not enough, show how you got it.) NOTE: Only those buttons which are ON are shown with an **X**, the OFF ones do not show any symbol.

X	X	X
X	X	X
X		

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X	X	X
X	X	X
		X

III. **(20 pts.)** This question refers to the "sliding tiles" toy we have studied in class. In each of the two situations shown the aim is to achieve the counterclockwise alphabetical arrangement partially shown in the figures (each figure has at most four tiles out of order.) Identify precisely which and how many pivot operations are needed in each case. As usual, be sure that:

(a) if you use just one pivot, identify it by showing the four tiles being pivoted.

(b) if you use a sequence of 17 pivots to change the parity, identify the first one as explained in **(a)**, and clearly identify the relevant result.

(c) if you use a sequence of 8 pivots to "rotate" three consecutive tiles, identify the direction of rotation and which tile is the zero-th tile.

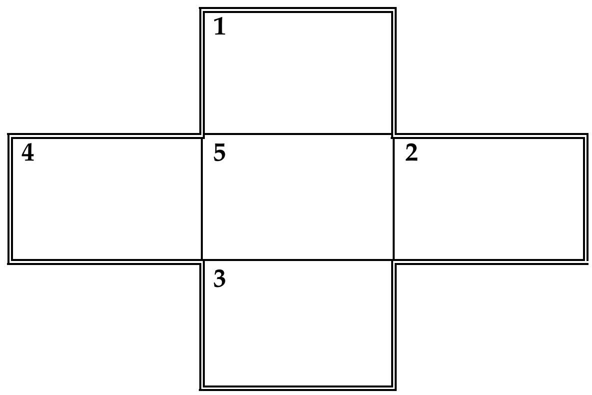
A. **(10 pts.)**

F	E	B	C	D	A	T	S	R
G								Q
H	I	J	K	L	M	N	O	P

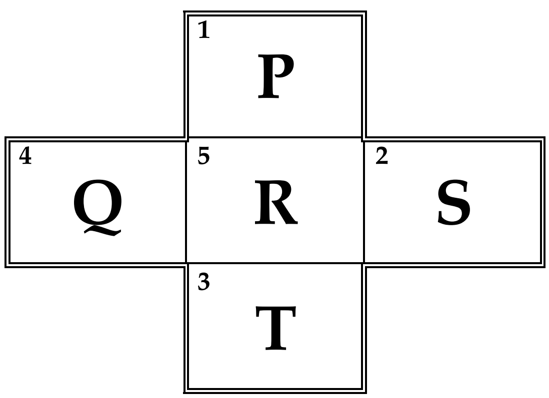
B. **(10 pts.)**

F	E	D	C	B	R	S	A	T
G								Q
H	I	J	K	L	M	N	O	P

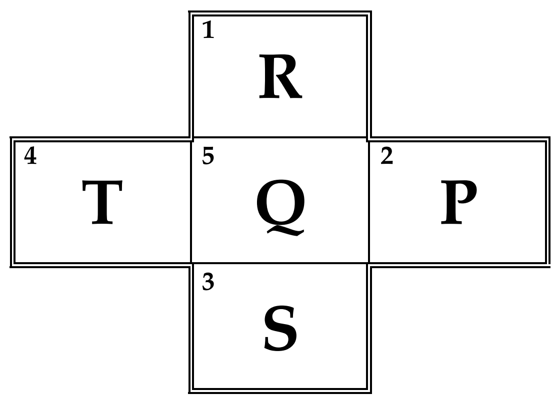
IV. (20 pts.) In the toy shown in the figure the contents of the center tile can be exchanged with any one of the four "external" tiles, that is, you can exchange the contents of tile 5 with the contents of tile 1, or tile 2, or tile 3, or tile 4.
No other one-step action is possible.



A. (10 pts.) Show that one can go from the START configuration to the TARGET configuration shown in the figure using the four legal one-step actions. Show the result of each one-step action as you apply it.



START



TARGET

(HINT: The cycle (12345) can also be written as (51234))

B. (10 pts.) Show that one can exchange the contents of tiles 1 and 4 using three of the possible one-step actions. Do the same for tiles 2 and 4. Is any arrangement of the four letters P, Q, R and S achievable? Why?

V. **(15 Pts.)** A 1-network (that is, consisting only of vertices and edges) is said to be complete if any two distinct vertices are joined by exactly one edge, and no vertex is joined to itself by an edge.

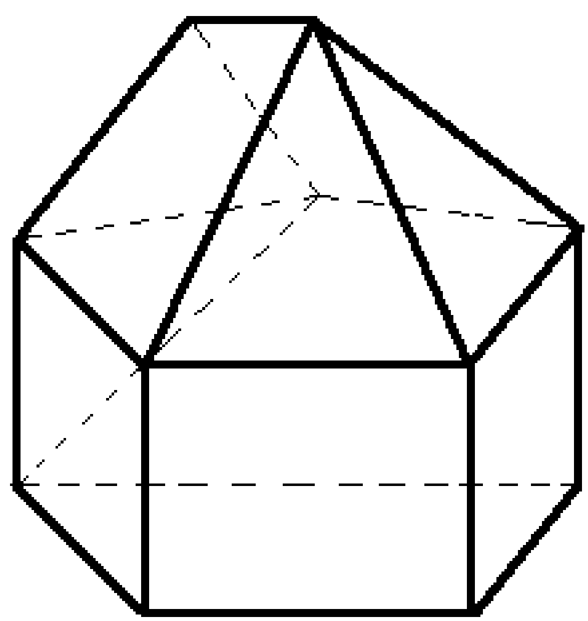
A. **(7 pts.)** Draw a complete 1-network with five vertices, and a complete 1-network with six vertices.

B. **(8 pts.)** Complete 1-networks are either eulerian or non-eulerian (a true, but not very deep statement!) Which is which, that is, can you tell which complete 1-networks are eulerians, and which ones are not?

VI. **(10 pts.)** Two 1-networks have the same number of vertices, and the same number of edges, however one is eulerian, the other one is not. Is this possible? If your answer is YES, show them to me, if NO, explain why.

VII. **(25 pts.)** This question deals with 2-networks (that is, consisting of vertices, edges and faces).

A. **(12 pts.)** Draw the planar 2-network representing the solid shown.in the figure.



Count precisely how many triangles, quadrangles, pentagons, etc, make up the faces, then verify Euler's formula for planar 2-networks.

B. **(13 pts.)** A planar 2-network is supposed to consist of exactly 9 vertices, 16 edges, three quadrangles, some triangles and some pentagons. If you can draw it, do it, if not, explain why you cannot.

VIII. (**20 pts.**) Shown below are three rankings of six alternatives **A, B, C, D, E** and **F**.

I	II	III
	A	**A**
A – B	**B**	**B – C**
C – D	**C**	**D**
E – F	**D – E – F**	**E – F**

A. **(10 pts.)** Which two rankings are in most agreement, according to the normal definition of distance as used in class?

B. **(10 pts.)** Which ranking is the most assertive, according to the normal definition of distance as used in class?