

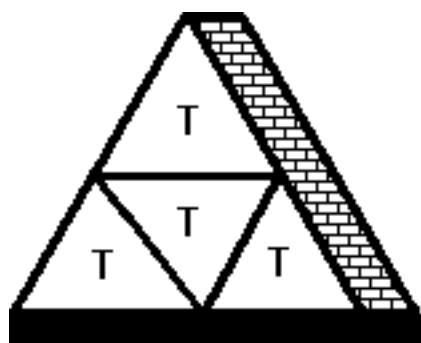
UNIVERSITY OF NOTRE DAME
DEPARTMENT OF MATHEMATICS

Math. 103 - Processes of Mathematical Thought. - Spring 1996 - Prof. Borelli
Exam I - February 28, 1996

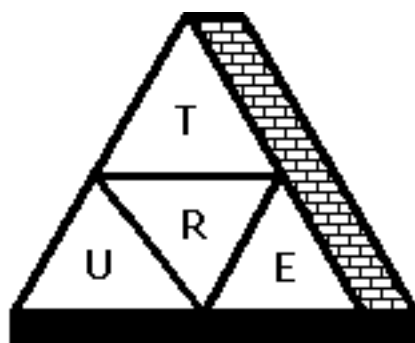
NOTE: This is an OPEN BOOK exam, **but** pocket calculators are not allowed, nor is collaboration among students. **You are under the University's Honor Code.**

1. The two figures below show the "START" and "TARGET" configurations of a game which is wired as follows:

Button	Acts on
Top	Lower Left, Center
Lower Left	Top, Lower Right
Lower Right	Lower Left, Lower Right, Center
Center	Top, Lower Left, Lower Right



START



TARGET

Each button can show one of the four letters E, R, T, U, and each action cycles through alphabetically, that is:

$$E \rightarrow R \rightarrow T \rightarrow U \rightarrow E$$

- A. (8 pts.) Decide what the "modulus" is for this game, that is, how many times to push one button so that everything stays the same. Show the multiplication table for this modulus.
- B. (8 pts.) Set up the system whose solution solves this game.
- C. (16 pts.) Solve the system using the Gauss-Jordan elimination method shown in class.
- D. (10 pts.) Solve the game as shown, that is, show a sequence of buttons which, when pushed, will take you from the "START" shown to the "TARGET" shown.
- E. (8 pts.) Verify with a sequence of pictures that your solution is correct

2. Let **Peter** and **Sam** be the two permutations on eight symbols shown below:

$$\mathbf{Peter} = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 8 & 6 & 4 & 3 & 1 & 2 & 7 \end{vmatrix} \quad \mathbf{Sam} = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 5 & 3 & 6 & 4 & 7 & 8 & 1 \end{vmatrix}$$

- A. (10 pts.) Write **Peter**•**Sam** as a product of disjoint cycles.
- B. (10 pts.) Write **Sam**•**Peter** as a product of transpositions.
- C. (10 pts.) Write the reverse of **Sam** as a product of disjoint cycles.
3. Let **Vanessa** = (23517)(43518)(2416)
- A. (5 pts.) Write **Vanessa** as a product of eleven transpositions.
- B. (10 pts.) Write **Vanessa** as a product of disjoint cycles.
- C. (5 pts.) Write **Vanessa** as a product of seven transpositions.