## Math. 103 - Processes of Mathematical Thought Exam No. 2 - March 27, 1996 - Prof. Borelli

**<u>NOTE</u>**: This is an OPEN BOOK exam. You are under the Honor Code, so that the work you hand in must be your own. Please try to show all of your work, so that I can assess the extent of your knowledge and give partial credit when appropriate.

**1.** Show below are two configurations of the "roadtoy." Assume that all the ten letters not shown are already properly alphabetized.



- A. (15 pts.) Identify precisely which eight pivots will alphabetize Configuration no. 1.
- **B.** (25 pts.) State how many pivota are needed to alphabetize Configuration no. 2. Explain your answer and identify precisely the last eight pivots you plan to use.
- 2. Consider the toy shown below, which consists of a "box" sitting on a pedestal, with four tiles on which the letters A, H, M, T appear, initially at random.



The possible moves are:

- Interchange any two adjacent tiles, but <u>not</u> two diagonally opposite tiles.
- Rotate the whole box 90° degrees on its pedestal in either direction.
- A. (10 pts.) Compute (12)(1234)(14) and (23)(1234)(12).
- **B.** (10 pts.) Now show that the toy allows you to achieve the interchange of any two tiles you wish.

**C.** (10 pts.) Determine which sequence of possible moves will take you from the initial configuration



to the final configuration



Show the effect of each one of the possible moves in the sequence.

**3.** Using the encoding permutation:

## (TAVUBXZMS)(EYPQOLHCK)(DARWINGFJ)

A. (18 pts.) decode the message:

 $\Delta CYVOBNKEVXWLIGVJLZVMNQTVSRAHPVRGAVDBSQT$ 

**B.** (12 pts.) encode the message

PLEASE KEEP JOY IN YOUR HEART