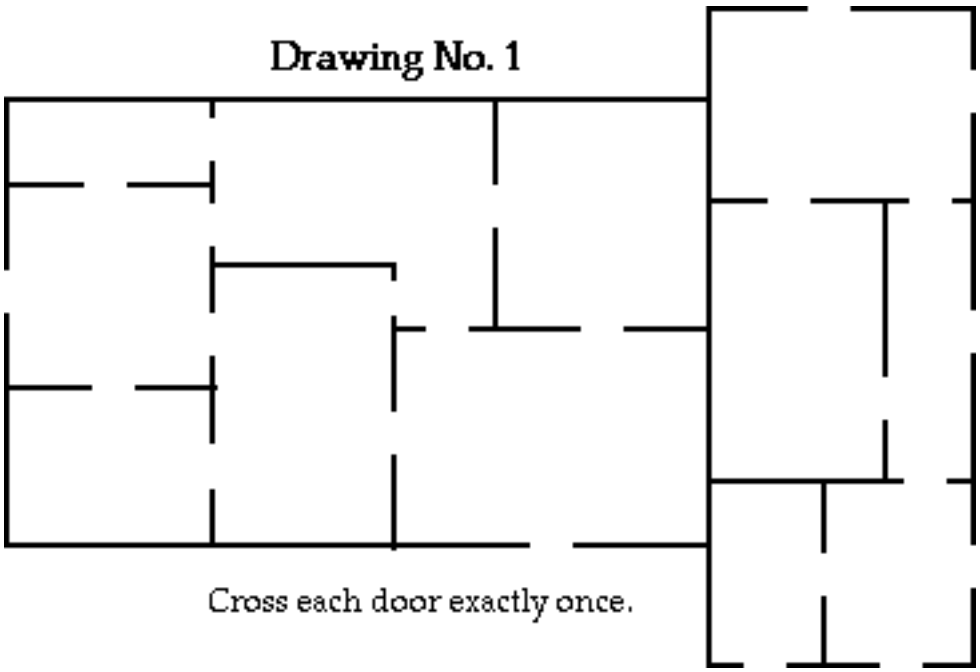
NAME _____

MATH. 103 - PROCESSES OF MATHEMATICAL THOUGHT

QUIZ NO. 3 *April 24, 1996*

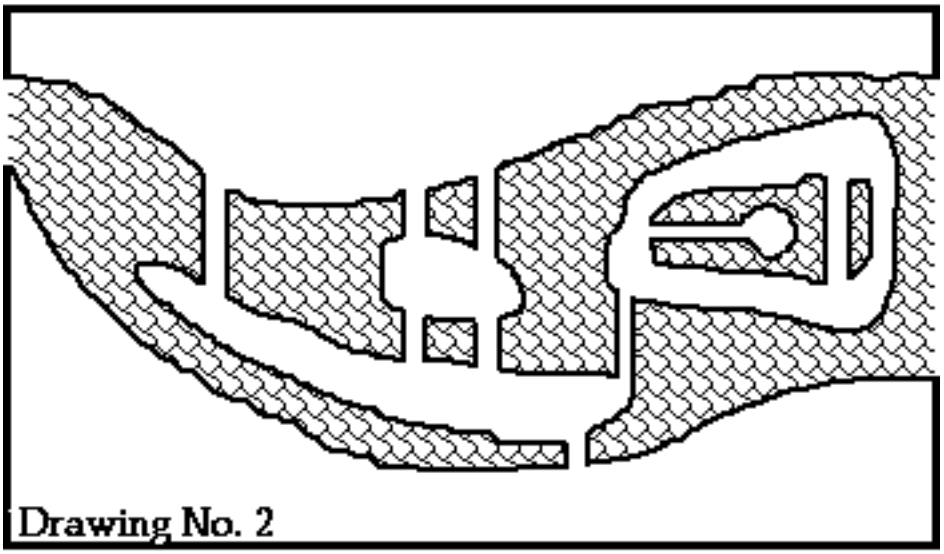
1. ***(14 pts.)*** Consider the two drawings shown below.

Drawing No. 1



Cross each door exactly once.

"A River Runs Through It." The larger island has a lake, with a smaller island inside it.



Drawing No. 2

 = Water. Cross each bridge exactly once.

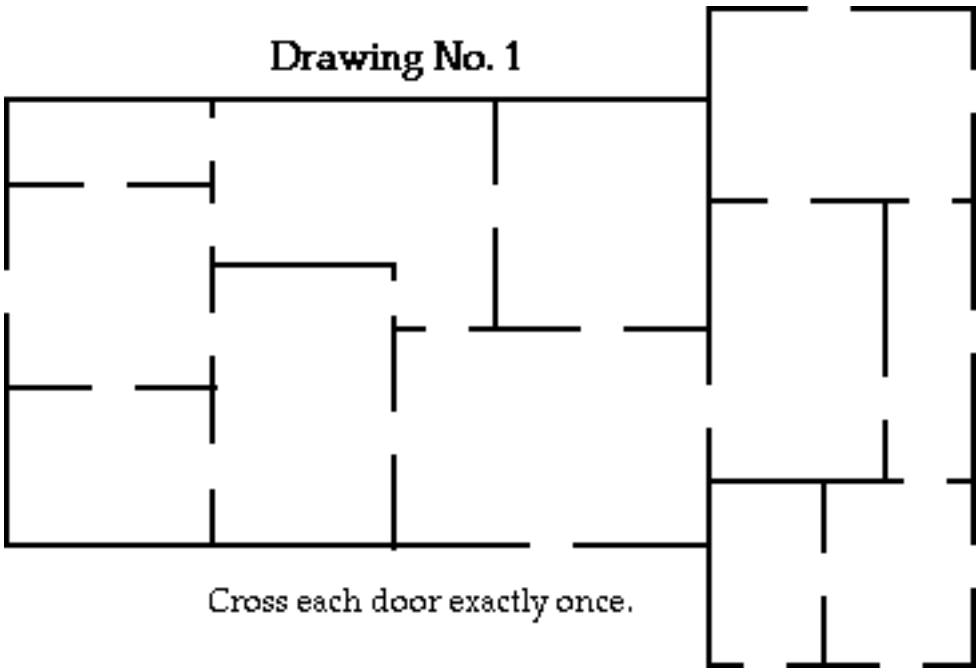
NAME _____

MATH. 103 - PROCESSES OF MATHEMATICAL THOUGHT

QUIZ NO. 3 *April 24, 1996*

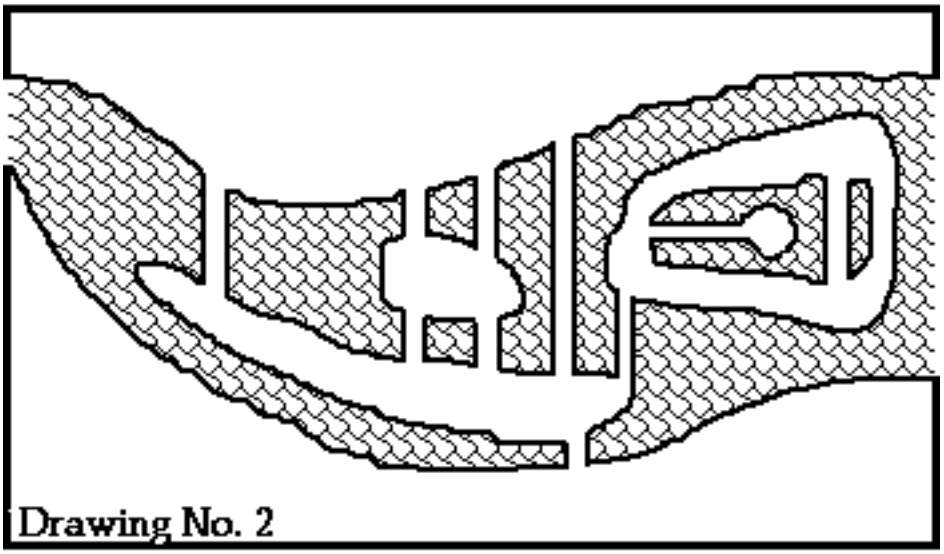
1. ***(25 pts.)*** Consider the two drawings shown below.

Drawing No. 1



Cross each door exactly once.

"A River Runs Through It." The larger island has a lake, with a smaller island inside it.



Drawing No. 2

 = Water. Cross each bridge exactly once.

A. *(8 pts.)* For each of the two drawings, draw the corresponding one-dimensional network.

B. *(6 pts.)* For each network, tell the number of odd vertices.

C. *(4 pts.)* One of the two networks is Eulerian, the other isn't. Decide which is which. Explain the reason for your answer.

D. *(7 pts.)* For that network which IS Eulerian, show **in the corresponding drawing** a Eulerian path.

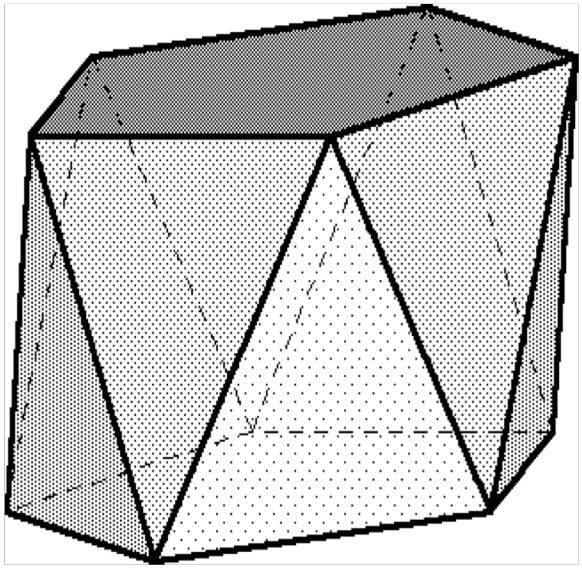
2. ***(15 pts.)*** Draw :

A. *(5 pts.)* A one-piece Eulerian network with no even vertices.

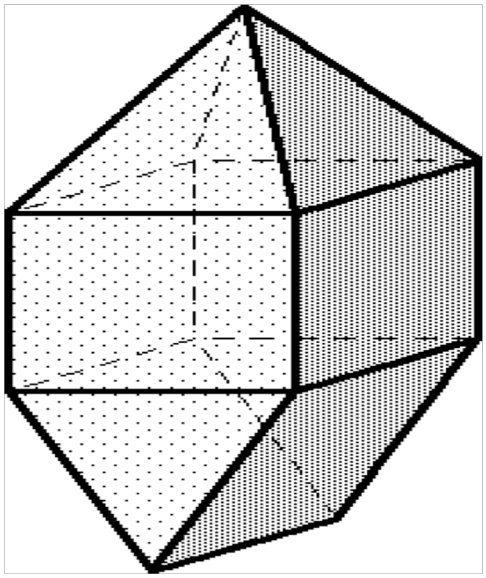
B. *(5 pts.)* A one-piece non-Eulerian network with no even vertices.

C. *(5 pts.)* A one-piece non-Eulerian network with no odd vertices.

3. ***(25 pts.)*** For each of the two solids shown below:



Solid No. 1



Solid No. 2

A. *(10 pts.)* Draw the corresponding planar network..

B. *(8 pts.)* State how many triangles, quadrilaterals, pentagons, etc. constitute the faces of each solid.

C. *(7 pts.)* Verify Euler's formula, by counting vertices, edges and faces.

4. ***(24 pts.)*** Consider the three solids described below. Draw the planar network of each one of the three solids, if possible.

A. *(8 pts.)* The faces consist of four triangles, three quadrilaterals, two pentagons.

B. *(8 pts.)* The faces consist of five triangles, three quadrilaterals, one pentagon.

C. *(8 pts.)* The faces consist of four triangles, two quadrilaterals, one septagon (7-sided.)

5. ***(11 points)*** Do **ONLY ONE** of the following two problems.

A. Two one-dimensional networks **M** and **N** are identical copies of each other. Network **P** is obtained from **N** by adding one edge joining two existing vertices A and B of **N**. It is known that **P** is eulerian and that **N** is not. How many odd vertices does **M** have? Justify your answer as completely as possible.

B. A one-dimensional network is called **"complete"** if every vertex is joined to every other vertex by exactly one edge. *(4 pts.)* Draw the complete network with 6 (six) vertices. Is it Eulerian? (6 pts.) Is the complete network with 28 vertices Eulerian? What about the complete network with 31 vertices, is it Eulerian? Justify your answers.

4.*(15 pts.)* Consider the three solids described below. Draw the planar network of each one of the three solids, if possible.

A. *(5 pts.)* The faces consist of five triangles, three quadrilaterals, one pentagon.

B. *(5 pts.)* The faces consist of five triangles, three quadrilaterals, one hexagon.

C. *(5 pts.)* The faces consist of five triangles, two quadrilaterals, one septagon (7-sided.)

5. ***(11 points)*** Do **ONLY ONE** of the following two problems.

A. Two one-dimensional networks **M** and **N** are identical copies of one and the same network. Network **P** is obtained from **N** by adding one edge joining two existing vertices A and B of **N**. It is known that **P** is not eulerian and that **N** is. How many odd vertices does **M** have? Justify your answer as completely as possible.

B. A one-dimensional network is called **"complete"** if every vertex is joined to every other vertex by exactly one edge. *(4 pts.)* Draw the complete network with 5 (six) vertices. Is it Eulerian? *(6 pts.)* Is the complete network with 27 vertices Eulerian? What about the complete network with 24 vertices, is it Eulerian? Justify your answers.