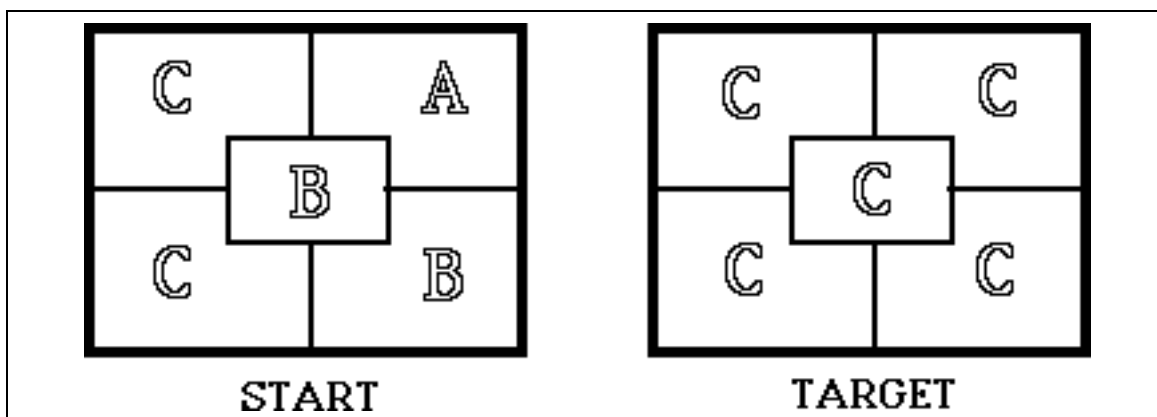# UNIVERSITY OF NOTRE DAME
DEPARTMENT OF MATHEMATICS

Math. 103 - Processes of Mathematical Thought. - Spring 1996 - Prof. Borelli
Final Examination - May 10 1996

> **NOTE:** This is an OPEN BOOK exam, **but** pocket calculators are not allowed, nor is collaboration among students. **You are under the University's Honor Code.**

1. The two figures below show the "START" and "TARGET" configurations of a game which is wired as follows:

Button	Acts on
Top Left	Lower Left, Center, Top Right
Top Right	Lower Right, Center, Top Left
Lower Left	Top Left, Top Right, Lower Right
Lower Right	Lower Left, Top Left, Top Right
Center	TopLeft, Top Right, Lower Left, Lower Right

Each button can show one of the three letters A, B, C, and each action cycles through alphabetically, that is:
$$A \longrightarrow B \longrightarrow C \longrightarrow A$$

A. **(7 pts.)** Decide what the "modulus" is for this game, that is, how many times to push one button so that everything stays the same. Show the multiplication table for this modulus.

B. **(8 pts.)** Set up the system whose solution solves this game. (Do NOT solve the system.)

C. **(10 pts.)** A partial "solution" of the game is shown in the next page. Use it, and some symmetry, to show a sequence of six buttons (some buttons may repeat) which, when pushed, will take you from the "START" shown to the "TARGET" shown.
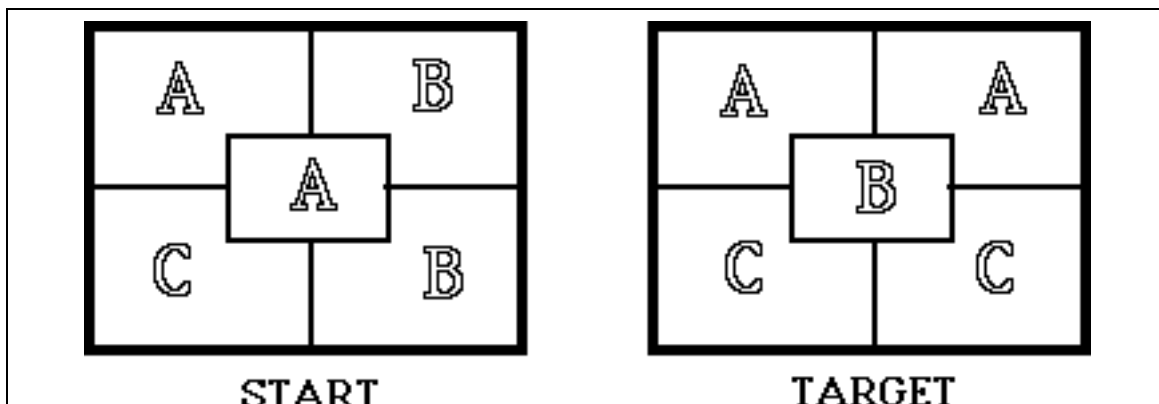
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A --> B --> C --> A

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To advance this button ONLY	Push these buttons
Top Left	Top Left, Top Right TWICE, Lower Right, Center
Lower Left	Lower Left TWICE, Center
Center	TopLeft TWICE, Top Right TWICE, Center

   D. **(5 pts.)** Verify with a sequence of pictures that your solution is correct

   E. **(10 pts.)** Suppose you are told that, for the game above, the TARGET is so bad that it takes the **maximum number of pushes** to get it. How many pushes is that? (Explain your answer)

2. Show below are two configurations of the "roadtoy."

Configuration no. 1   Configuration no. 2

   A. *(10 pts.)* State how many pivots are needed to alphabetize Configuration no. 1. Explain your answer and identify precisely the first pivot you plan to use.

   B. *(10 pts.)* Identify precisely which eight pivots will alphabetize Configuration no. 2.

3. Let **Peter** and **Sam** be two arbitrary permutations on eight symbols. Identify each of the statements below as **TRUE** or **FALSE**. In each case **explain your answer**.

   A. **(10 pts.) Peter•Peter** is always an even permutation.

   B. **(10 pts.) Sam•Sam•Sam** is always an odd permutation.

   C. **(10 pts.) Peter•Sam•Peter** and **Sam** always have the same <u>parity</u>.
   (they are either both even or both odd.)

4. Let **Vanessa** = (2645317)(75126)(12348765)
   A. **(5 pts.)** Is **Vanessa** even or odd? Explain your answer.
   B. **(10 pts.)** Write **Vanessa** as a product of disjoint cycles.
   C. **(5 pts.)** Write **Vanessa** in the two-row format, that is, fill the blanks on the next page:

UNIVERSITY OF NOTRE DAME
DEPARTMENT OF MATHEMATICS

To advance this button ONLY	Push these buttons
Top Left	Top Left, Top Right TWICE, Lower Right, Center
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D. (5 pts.) Verify with a sequence of pictures that your solution is correct

E. (10 pts.) Suppose you are told that, for the game above, the TARGET is so bad that it takes the **maximum number of pushes** to get it. How many pushes is that? (Explain your answer)

2. Show below are two configurations of the "roadtoy."

M	N	Q	P	O	R	S	T	A
L								B
K	J	I	H	G	F	E	D	C

Configuration no. 1

M	N	P	Q	O	R	S	T	A
L								B
K	J	I	H	G	F	E	D	C

Configuration no. 2

A. (15 pts.) State how many pivots are needed to alphabetize Configuration no. 1. Explain your answer and identify precisely the first pivot you plan to use.

B. (15 pts.) Identify precisely which eight pivots will alphabetize Configuration no. 2.

3. Let **Peter** and **Sam** be two arbitrary permutations on eight symbols. Identify each of the statements below as **TRUE** or **FALSE**. In each case **explain your answer**.

A. (10 pts.) Peter•Peter is always an even permutation.

B. (10 pts.) Sam•Sam•Sam is always an odd permutation.

C. (10 pts.) Peter•Sam•Peter and **Sam** always have the same <u>parity</u>. *(they are either both even or both odd.)*

4. Let **Vanessa** = (2645317)(75126)(12348765)

A. (5 pts.) Is **Vanessa** even or odd? Explain your answer.

B. (10 pts.) Write **Vanessa** as a product of disjoint cycles.

C. (10 pts.) Write **Vanessa** in the two-row format, that is, fill the blanks on the next page:

$$\text{Vanessa} = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \underline{} & \underline{} & \underline{} & \underline{} & \underline{} & \underline{} & \underline{} & \underline{} \end{vmatrix}$$

5. "My home has exactly three doors leading outside, and no room in my home has more than two doors. At night I am able to walk from room to room (and outside), locking each door as I go through it, then go to bed." Given that the preceding statement is true, which of the following four conclusions **must** necessarily follow? Explain each of your answers.

    A.   **(10 pts.)** Every room in my home has exactly two doors.

    B.   **(10 pts.)** My bedroom has only one door.

    C.   **(10 pts.)** I start my nightly walk in my bedroom.

    D.   **(10 pts.)** I start my nightly walk outdoors.

6. Only one of the four solids described below exists. Decide which is which and explain your anwers. For the one which does exist draw the planar network of it.

    A.   **(10 pts.)** The faces of the solid are exactly

                one octagon (8 sides), four triangles.

    B.   **(10 pts.)** The faces of the solid are exactly

                one octagon (8 sides), three triangles, one quadrilateral

    C.   **(10 pts.)** The faces of the solid are exactly

                one octagon (8 sides), two triangles, one quadrilateral

    D.   **(10 pts.)** The faces of the solid are exactly

                one octagon (8 sides), two triangles, one hexagon (6 sides).

7. Three political commentators, Mr. Smith, Ms, Toth and Mr. Upton, have been asked by a localnewspaper to rank their preferences of the five candidates Adams, Brown, Collins, Davis and Eaton for the position of dog-catcher. Shown below are their responses.

Mr. Smith	Ms. Toth	Mr. Upton
3 Adams	2 Adams	5 Adams
2 Brown	4 Brown	2 Brown
5 Collins	1 Collins	3 Collins
2 Davis	4 Davis	1 Davis
4 Eaton	2 Eaton	4 Eaton

    A.   **(10 pts.)** Construct the ranking triangle for each commentator..
    B.   **(10 pts.)** Which two are most in agreement?.
    C.   **(10 pts.)** Which two are most in disagreement?.