**NOTE:** This is an **OPEN book** exam. This means you **MAY** use any notes, books, pocket calculators or any other learning aids you have brought with you. What you **MAY NOT** use is the brain of people sitting around you. <u>You are under the University's Honor Code</u>, and therefore are pledged to hand in work which is entirely your own. Speaking of work, make sure you <u>show all of your work,</u> since questions will be graded with the possibility of earning partial credit; in addition, the instructor tends to look incredulously at correct answers without any work showing how they have been obtained. Finally, <u>hand in all the exam sheets in the blue booklet.</u>

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In questions 1, 2 and 3 the term "good action" refers to those actions which change exactly one button, exactly one step in the sequence of stages (e.g. On/Off, or Off/Dim/Bright, etc.) Good actions are denoted by G's. The term "bad actions" refers to those actions, provided by the game, which change more than one button at the same time. Bad actions are denoted by S's.

1. *(20 pts.)* This question refers to the "original" version of the nine-button game, the version we studied first in class. Shown below are a "starting" configuration and a "target" configuration. Give the sequence of bad actions which must be applied to reach the target from the starting configuration. No bad action is allowed to be applied twice.
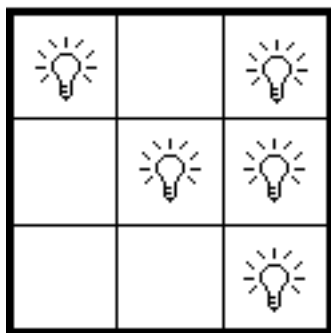
Starting   Target
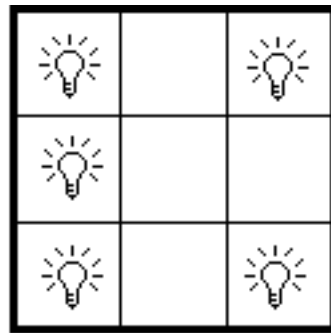
**2.** *(30 pts.)* This question refers to a **new** nine-button, On/Off game, about which the following is given:

    i.   the game is symmetric.

    ii.   the table shown below holds.

TO ACHIEVE GOOD ACTION	APPLY BAD ACTIONS
G_1	S_1 S_2 S_4 S_5 S_9
G_2	S_4 S_6 S_8
G_3	
G_4	
G_5	S_1 S_2 S_3 S_4 S_6 S_7 S_8 S_9
G_6	
G_7	
G_8	
G_9	

  A. *(15 pts.)* Complete the table.

  S. *(15 pts.)* Shown on the next page are a "starting" configuration and a "target" configuration. Give the sequence of bad actions which must be applied to reach the target from the starting configuration. No bad action is allowed to be applied twice.
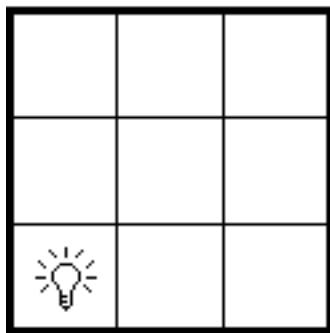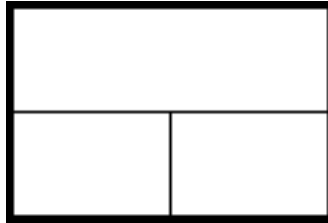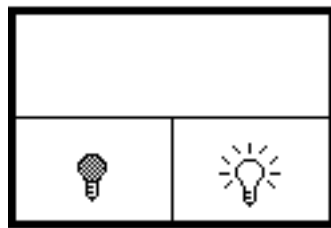
Starting
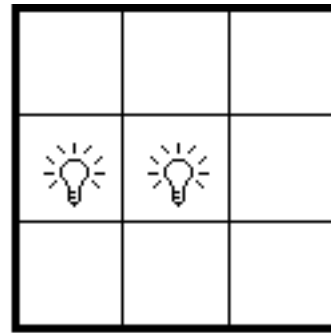
Target

3. *(30 pts.)* The toy shown below consists of three buttons, one large and two small, and is wired as follows:


The large button affects all three buttons. Each one of the two small buttons affects itself and the large button (but NOT the other small button.) Each button goes through the sequence

OFF ---> DIM ---> BRIGHT ---> OFF

A. *(10 pts.)* Write the system which expresses each bad action in terms of the three good actions.

S. *(10 pts.)* Solve the system using the method we have learned in class (or any other method you devise, but ...) SHOW YOUR WORK in the grid sheets provided.

C. *(10 pts.)* Shown below are a "starting" configuration and a "target" configuration for the toy. (The shaded bulb stands for DIM.) Give the sequence of bad actions which must be applied to reach the target from the starting configuration. No bad action is allowed to be applied three times, but may have to be applied twice.
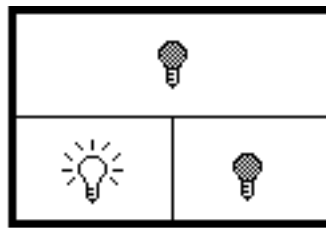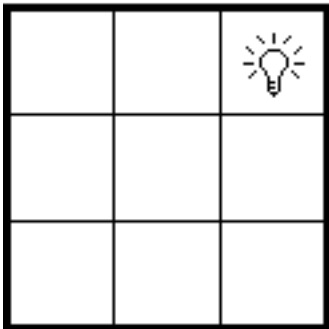

Starting   Target
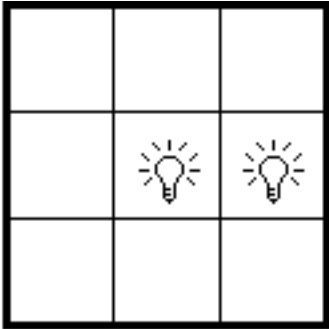
4. *(20 pts.)* Consider the two permutations:

$$\text{Peter} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 4 & 6 & 7 & 1 & 2 \end{bmatrix} \quad \text{and} \quad \text{Sally} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 2 & 1 & 7 & 3 \end{bmatrix}$$

A. *(8 pts.)* Compute both compositions   Peter•Sally   and   Sally•Peter.

S. *(5 pts.)* Write Peter in cycle notation.

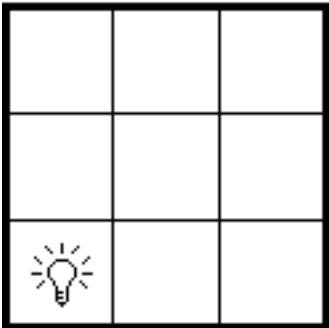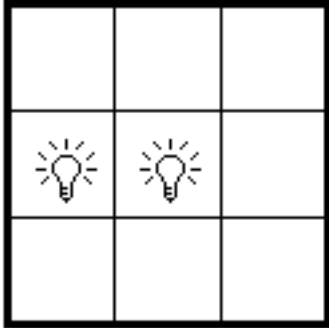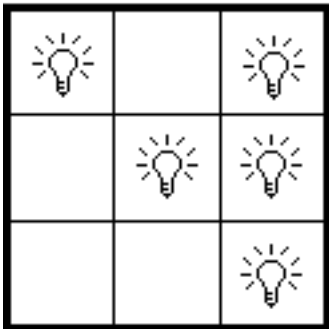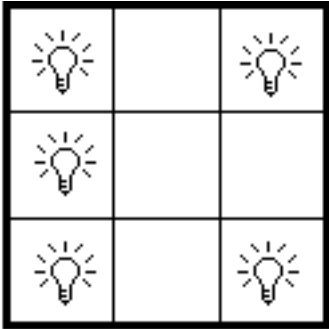**C.** *(7 pts.)* Write the inverse of Sally in cycle notation.

Starting

Target

Starting

Target

Starting

Target