

MATH. 103 - PROCESSES OF MATHEMATICAL THOUGHT
QUIZ NO. 2

March 24 1997

NOTE: This is an OPEN BOOK exam. You are under the Honor Code, so that the work you hand in must be your own. Please try to show all of your work, so that I can assess the extent of your knowledge and give partial credit when appropriate.

1. (20 pts.) Consider the permutation on seven symbols

$$(2\ 3\ 7\ 5)(4\ 6\ 1)(5\ 3\ 2\ 7)(3\ 4)$$

- A. (3 pts.) Is the permutation even or odd?
- B. (6 pts.) Write the permutation as a sequence of nine transpositions.
- C. (8 pts.) Write the permutation as a sequence of five transpositions
- D. (3 pts.) Write the permutation as a sequence of six transpositions.
2. (40 pts.) Shown below are two configurations of the “roadtoy.” Assume that all the ten letters not shown are already properly alphabetized. When you use the seven-letter configuration to identify pivots, state precisely which is the “zero-th” letter.

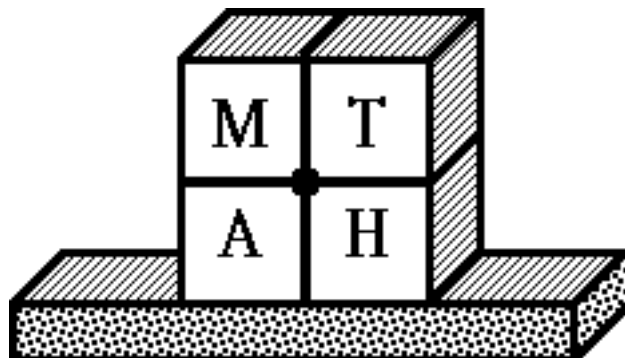
L	M	N	O	P	Q	T	R	S	A
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Configuration no. 1

L	M	N	O	P	S	T	R	Q	A
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Configuration no. 2

- A. (15 pts.) Identify precisely which eight pivots will alphabetize Configuration no. 1.
- B. (25 pts.) State how many pivots are needed to alphabetize Configuration no. 2. Explain your answer and identify precisely the last eight pivots you plan to use.
3. (30 pts.) Consider the toy shown below, which consists of a “box” sitting on a pedestal, with four tiles on which the letters A, H, M, T appear, initially at random. (Count the tiles starting with where the “M” is and going clockwise.)



(3 continued.)

The possible moves are:

- Interchange any two adjacent tiles, but **not** two diagonally opposite tiles.
- Rotate the whole box 90° degrees on its pedestal in either direction.

- A. (10 pts.) Compute $(12)(1234)(14)$ and $(23)(1234)(12)$.
- B. (10 pts.) Now show that the toy allows you to achieve the interchange of any two tiles you wish.
- C. (10 pts.) Next, convince me that one can achieve any permutation of the four letter one please, using one of the results shown in class.
4. (10 pts.) (An application to cryptography.) The message appearing below

MXCWXMLMVFXRH XORUQ

has been received. It was sent by encoding the original message according to the permutation (for example, the "M" in the message comes from an "T")

(ACNVZDFHJ) (QKLORSUPΔXYT) (WEBGIM)
(The symbol "Δ" represents a *blank space*.)

- A. (6 pts.) What was the original message?
- B. (4 pts.) You want to answer

YO TAMBIEN

using the same code. What do you send?