NAME _____

MATH. 103 - PROCESSES OF MATHEMATICAL THOUGHT

QUIZ NO. 3 *April 23, 1997*

1. ***(22 pts.)*** Consider the two maps shown below:

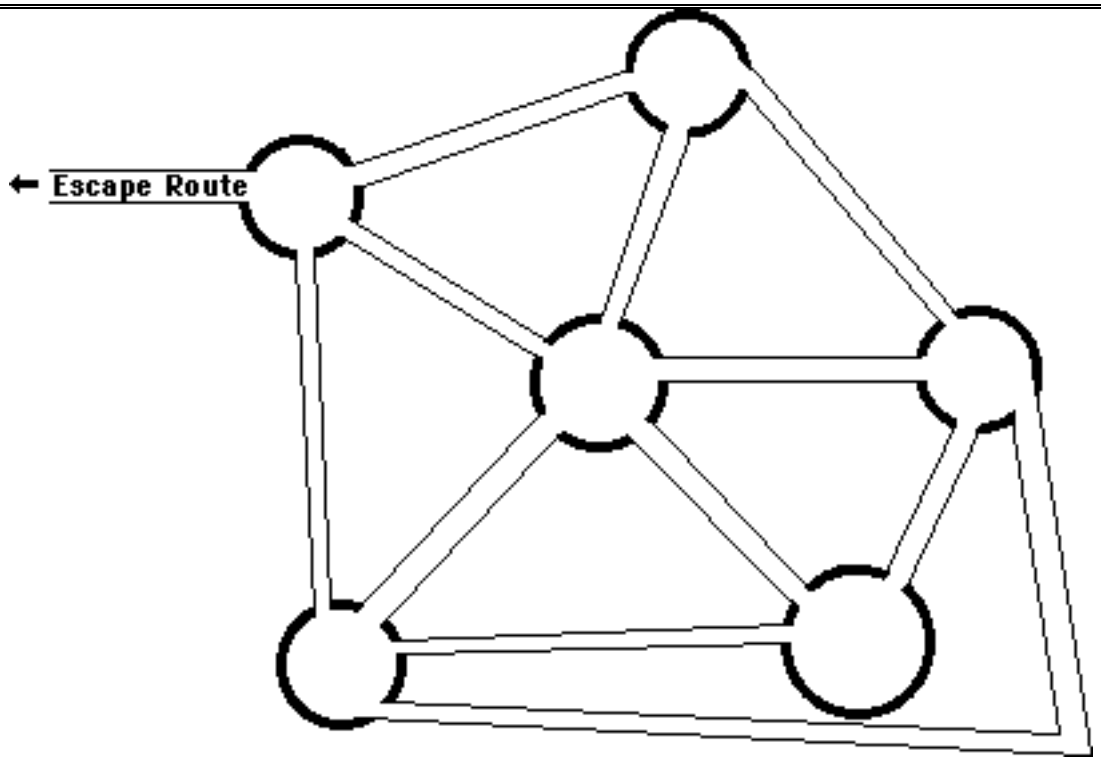

Figure 1

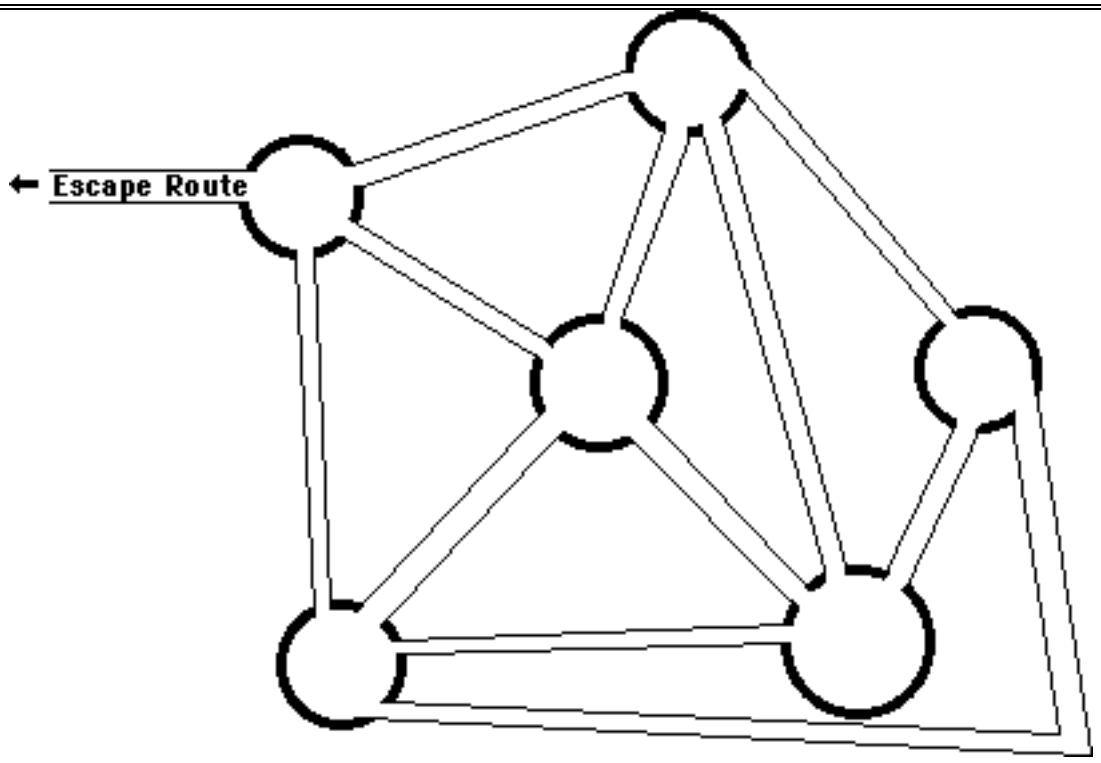

Figure 2

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QUIZ NO. 3 *April 23, 1997*

1. ***(22 pts.)*** Consider the two maps shown below:

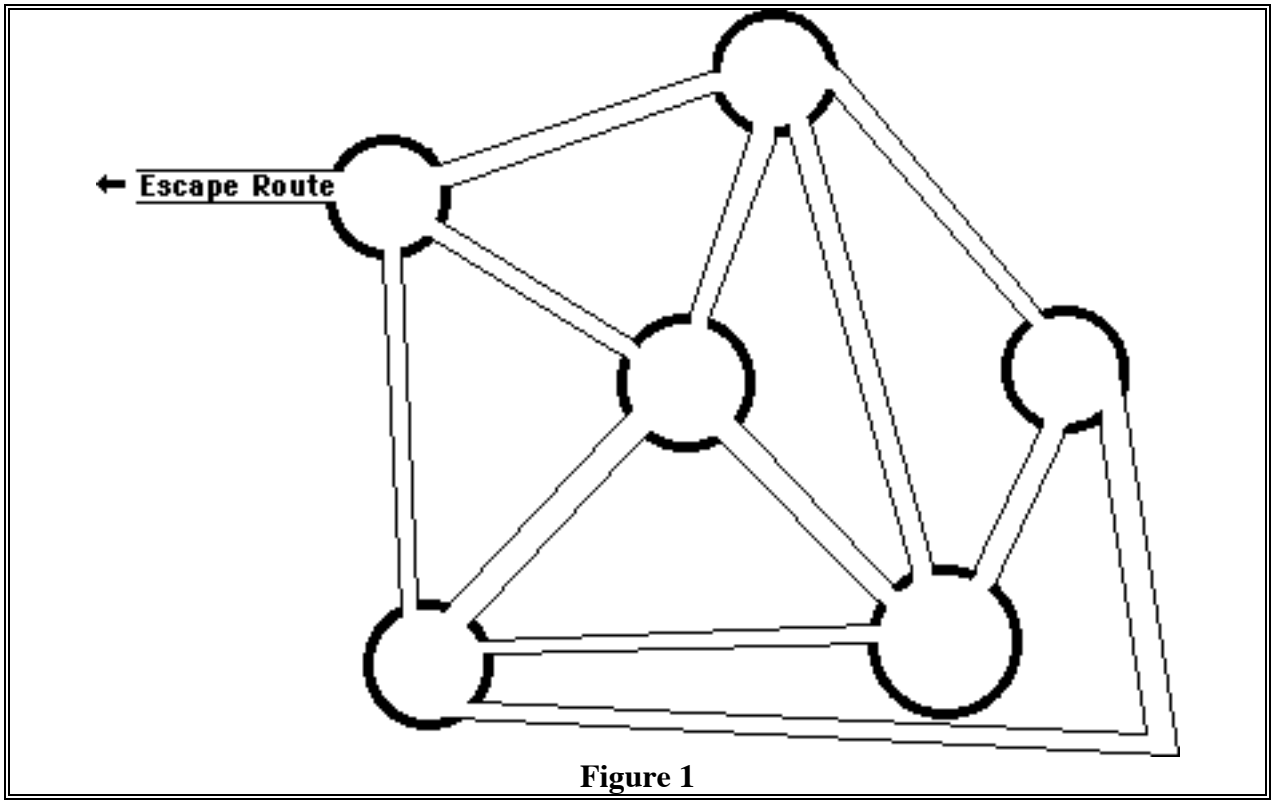


Figure 1

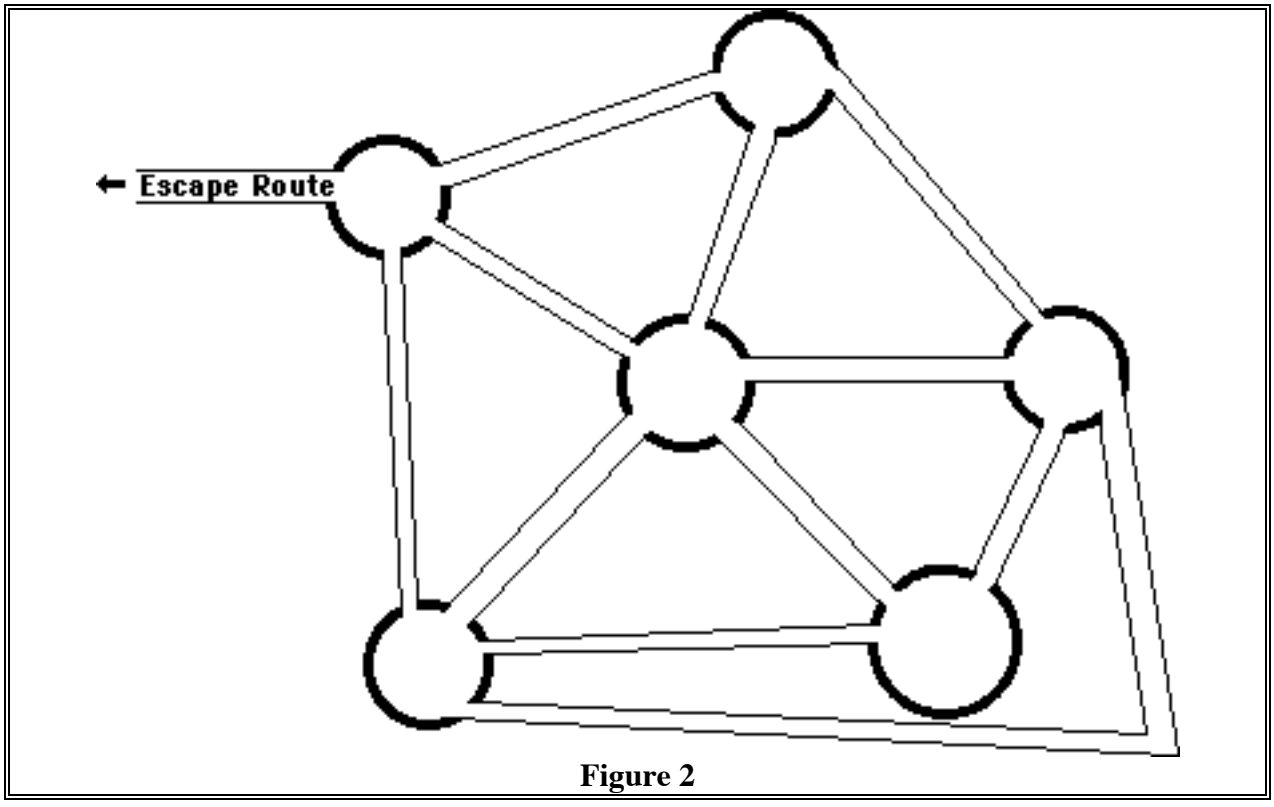


Figure 2

You are General S_oR_e L. Oozer of the Carpassian Army, and you have been ordered to mine all roads in the two maps shown in the previous page. (heavy circles represent settlements) before following the Escape Route indicated. Of course, you do NOT want to use a road you have just mined, it may blow you up, and you do NOT want to mine your escape route!

A. *(6 pts.)* For each of the two maps, draw the corresponding one-dimensional network. Clearly label the vertices.

B. *(6 pts.)* For each network, tell the number of odd vertices. (Read the question carefully!)

C. *(4 pts.)* In one of the maps your mission is achievable, in the other it isn't. Decide which is which. Explain the reason for your answer.

D. *(6 pts.)* For that map where you can accomplish your mission, show **in the map** the path of your mine-layer.

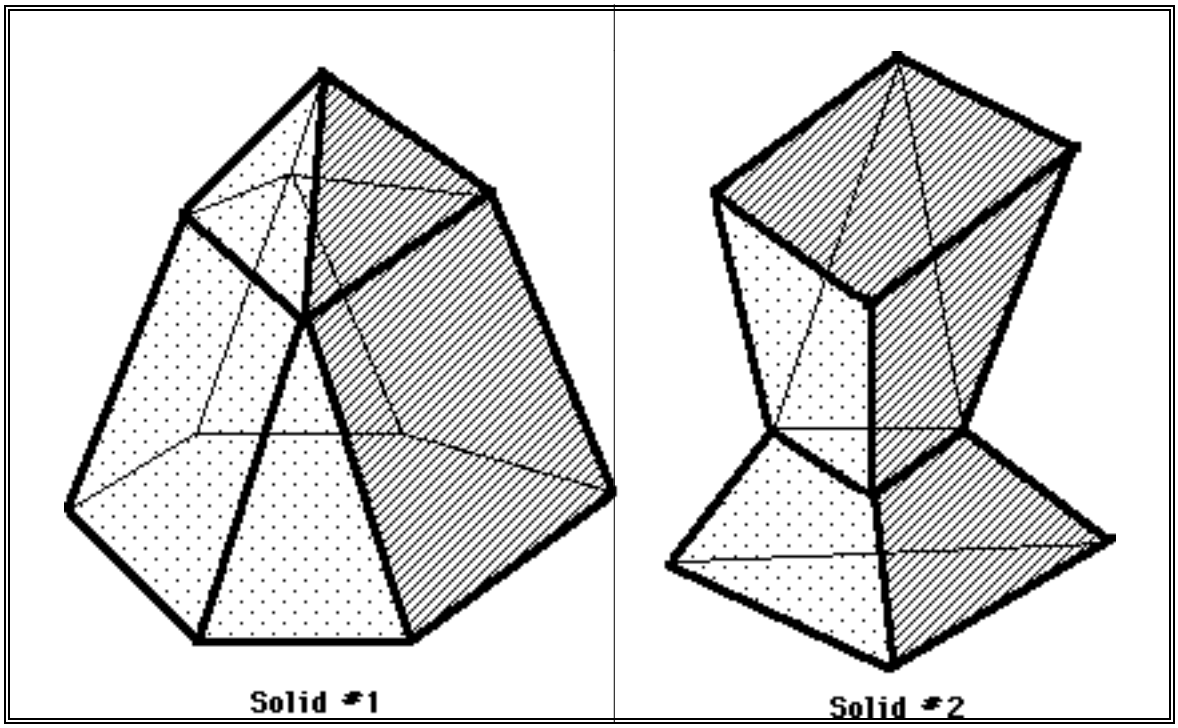
2. ***(15 pts.)*** If possible, draw the following. If not possible, explain why.

A. *(5 pts.)* A one-piece Eulerian network with no even vertices and at least two edges.

B. *(5 pts.)* A one-piece non-Eulerian network with exactly three vertices.

C. *(5 pts.)* A one-piece non-Eulerian network with no odd vertices.

3. ***(25 pts.)*** For each of the two solids shown below:



A. *(10 pts.)* Draw the corresponding planar network. Be sure to label the vertices on both the solid and the planar network.

B. *(8 pts.)* State how many triangles, quadrilaterals, pentagons, hexagons, etc. constitute the faces of each solid.

C. *(7 pts.)* Verify Euler's formula, by counting vertices, edges and faces.

4. ***(20 pts.)*** Consider the two solids described below. Draw the planar network of each one of the two solids, if possible.

A. *(10 pts.)* The faces consist of four triangles, three quadrilaterals, two pentagons.

B. *(10 pts.)* The faces consist of four triangles, two quadrilaterals, one septagon (7-sided.)

5. ***(18 points)*** Do **ONLY ONE** of the following two problems.

A. Two 1-networks **M** and **N** are identical copies of each other. Network **P** is obtained from **N** by adding one edge joining two existing vertices A and B of **N**. It is known that **P** is eulerian and that **N** is not. How many odd vertices does **M** have? Justify your answer as completely as possible.

B. A one-dimensional network is called **"complete"** if every vertex is joined to every other vertex by exactly one edge. *(8 pts.)* Draw the complete network with 6 (six) vertices. Is it Eulerian? *(10 pts.)* Is the complete network with 28 vertices Eulerian? What about the complete network with 31 vertices, is it Eulerian? Justify your answers.

4. ***(20 pts.)*** Consider the two solids described below. Draw the planar network of each one of the two solids, if possible.

A. *(10 pts.)* The faces consist of five triangles, three quadrilaterals, one hexagon.

B. *(10 pts.)* The faces consist of five triangles, two quadrilaterals, one septagon (7-sided.)

5. ***(18 points)*** Do **ONLY ONE** of the following two problems.

A. Two one-dimensional networks **M** and **N** are identical copies of one and the same network. Network **P** is obtained from **N** by adding one edge joining two existing vertices A and B of **N**. It is known that **P** is not eulerian and that **N** is. How many odd vertices does **M** have? Justify your answer as completely as possible.

B. A one-dimensional network is called **"complete"** if every vertex is joined to every other vertex by exactly one edge. *(8 pts.)* Draw the complete network with 7 (seven) vertices. Is it Eulerian? *(10 pts.)* Is the complete network with 27 vertices Eulerian? What about the complete network with 24 vertices, is it Eulerian? Justify your answers.