UNIVERSITY OF NOTRE DAME
DEPARTMENT OF MATHEMATICS

NAME (Please PRINT) ________________________________

MATH. 103 - PROCESSES OF MATHEMATICAL THOUGHT. - SPRING 1997

INSTRUCTOR: ______________________________

FINAL EXAMINATION - MAY 5, 1997

NOTE: This is an OPEN BOOK exam, **but** pocket calculators are not allowed, nor is collaboration among students.

<u>You are under the University's Honor Code.</u>

READ CAREFULLY:

1. Make sure you have 3 (three) distinct pages of questions (EXCLUDING this cover page.)
2. Write (PRINT) your name above and sign your name in the Attendance sheet.
3. Fill in (PRINT) the Instructor's name.
4. Write (PRINT) your name on the blue book cover.
5. Hand in this cover sheet and the three pages (please do NOT unstaple) inside the blue book.
6. Following correctly the five instructions given above is worth **5 (five)** points.

1. Questions **A** and **B** of this part **1** refer to the nine-button toy we have considered in class, with the buttons numbered in the usual manner shown below

1	2	3
4	5	6
7	8	9

A. **(6 pts.)** This question refers to the usual wiring of the toy, the one we studied first. Assume that in the initial configuration the buttons which are lit are exactly buttons # 2, 4 and 9. Your target configuration is the usual one, in which all buttons but # 5 are lit. Write down a sequence of buttons, without repetitions, which will take you from the initial to the target configuration.

B. **(8 pts.)** In this question the wiring of the toy is described as follows:

- the wiring is symmetric on the square.
- button # 1 acts on buttons # 1, 2, 3, 4, 7
- button # 2 acts on buttons # 4, 6, 7, 8, 9
- button # 5 acts on buttons # 1, 3, 5, 7, 9

Set up the system whose solution gives the needed answers for this game. **(Do NOT attempt to solve the system.)**

C. **(8 pts.)** Tell me how to modify the toy of question **B** so that the 'modulus' changes from 2 to 7. Then show me the first step in solving the system you set up previously, with this new modulus of 7.

D. **(8 pts.)** Suppose you are told that, for the toy of question **C**, the target is so bad that it takes at least the **maximum number of pushes** to get it. How many pushes is that? (Explain your answer)

1. Questions **A** and **B** of this part **1** refer to the nine-button toy we have con-sidered in class, with the buttons numbered in the usual manner shown below

1	2	3
4	5	6
7	8	9

A. **(6 pts.)** This question refers to the usual wiring of the toy, the one we studied first. Assume that in the initial configuration the buttons which are lit are exactly buttons # 1, 6 and 8. Your target configuration is the usual one, in which all buttons but # 5 are lit. Write down a sequence of buttons, without repetitions, which will take you from the initial to the target configuration.

B. **(8 pts.)** In this question the wiring of the toy is described as follows:

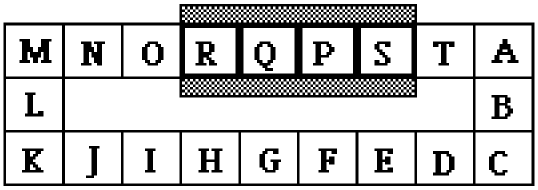
- the wiring is symmetric on the square.
- button # 1 acts on buttons # 1, 2, 4, 5, 9
- button # 2 acts on buttons # 1, 3, 4, 5, 6
- button # 5 acts on buttons # 1, 3, 5, 7, 9

Set up the system whose solution gives the needed answers for this game. **(Do NOT attempt to solve the system.)**

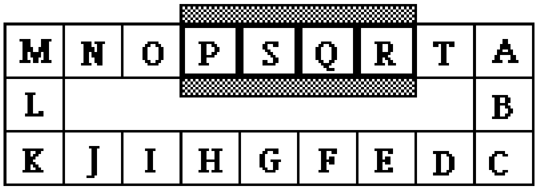
C. **(8 pts.)** Tell me how to modify the toy of question **B** so that the 'modulus' changes from 2 to 6. Then show me the first step in solving the system you set up previously, with this new modulus of 6.

D. **(8 pts.)** Suppose you are told that, for the toy of question **C**, the target is so bad that it takes the <u>**maximum number of pushes**</u> to get it. How many pushes is that? (Explain your answer)

2. Shown below are two configurations of the "roadtoy."



Configuration no. 1



Configuration no. 2

A. **(10 pts.)** State how many pivots are needed to alphabetize Configuration no. 1. Explain your answer and identify precisely the first pivot you plan to use.

B. **(15 pts.)** State how many pivots are needed to alphabetize Configuration no. 2. Explain your answer and identify precisely the last eight pivots you plan to use.

3. Let **Peter** and **Sam** be two arbitrary permutations on eight symbols. Identify each of the statements below as **TRUE** or **FALSE**. In each case **explain your answer**.

A. **(6 pts.) Peter•Peter** is always an even permutation.

B. **(6 pts.) Sam•Sam•Sam** is always an odd permutation.

C. **(6 pts.) Peter•Sam•Peter** and **Sam** always have different parities.

D. **(6 pts.) Sam•Peter** can always be written using disjoint cycles.

E. **(6 pts.) Sam•Peter** can always be written using at most seven transpositions

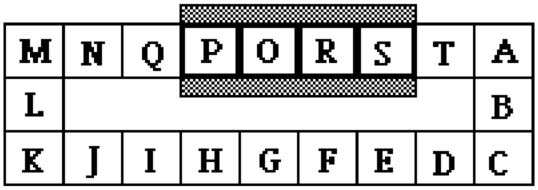
4. Let **Vanessa** = (3 5 1 9 6 8 7) (7 5 1 2 6 4 3 9) (1 2 3 4 8 7 9 6 5)

A. **(6 pts.)** Is **Vanessa** even or odd? Explain your answer.

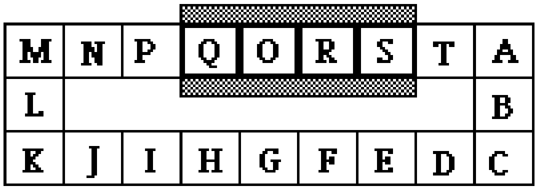
B. **(8 pts.)** Write **Vanessa** as a product of disjoint cycles.

C. **(6 pts.)** Write **Vanessa** in the two-row format, that is, fill the blanks on the next page:

2. Show below are two configurations of the "roadtoy."



Configuration no. 1



Configuration no. 2

A. **(10 pts.)** State how many pivots are needed to alphabetize Configuration no. 1. Explain your answer and identify precisely the first pivot you plan to use.

B. **(15 pts.)** State how many pivots are needed to alphabetize Configuration no. 2. Explain your answer and identify precisely the last eight pivots you plan to use.

3. Let **Peter** and **Sam** be two arbitrary permutations on eight symbols. Identify each of the statements below as **TRUE** or **FALSE**. In each case **explain your answer**.

A. **(6 pts.) Peter•Peter** is always an even permutation.

B. **(6 pts.) Sam•Sam•Sam** is always an odd permutation.

C. **(6 pts.) Peter•Sam•Peter** and **Sam** always have the same parity.

D. **(6 pts.) Sam•Peter** can always be written using disjoint cycles.

E. **(6 pts.) Sam•Peter** can always be written using at most seven transpositions

4. Let **Vanessa** = (3 5 1 7 6 9 8 4) (7 5 1 2 6 4 3 9) (1 2 3 4 8 7 9 6 5)

A. **(5 pts.)** Is **Vanessa** even or odd? Explain your answer.

B. **(7 pts.)** Write **Vanessa** as a product of disjoint cycles.

C. **(5 pts.)** Write **Vanessa** in the two-row format, that is, fill the blanks on the next page:

$$\textbf{Vanessa} = \left|\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \\ \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ & \_\_ \end{array}\right|$$

5. "My home has exactly three doors leading outside, and no room in my home has more than two doors. At night I am able to walk from room to room (and outside), locking each door as I go through it, then go to bed." Given that the preceding statement in quotes is true, which one(s) among the following four conclusions **must** necessarily follow? Explain each of your answers, (for every statement, tell me why it **need not follow**, or it **must follow** the premise.)

- **A.** **(6 pts.)** Every room in my home has exactly two doors.
- **B.** **(6 pts.)** My bedroom has exactly one door.
- **C.** **(6 pts.)** I start my nightly walk in my bedroom.
- **D.** **(6 pts.)** I start my nightly walk outdoors.

6. Only one of the four planar networks below exists. Decide which is which and explain your anwers. For the one which does exist, draw it.

- **A.** **(6 pts.)** The faces in the planar network are exactly
  one octagon (8 sides), four triangles.
- **B.** **(6 pts.)** The faces in the planar network are exactly
  one octagon (8 sides), three triangles, one quadrilateral
- **C.** **(6 pts.)** The faces in the planar network are exactly
  one octagon (8 sides), two triangles, one quadrilateral
- **D.** **(6 pts.)** The faces in the planar network are exactly
  one octagon (8 sides), two triangles, one hexagon (6 sides).