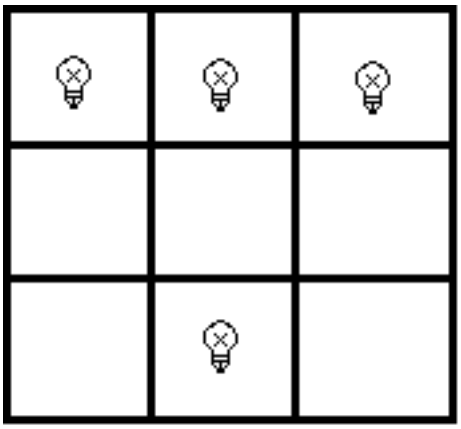
Math. 103 - Processes of Mathematical Thought
Prof. Mario Borelli
Exam I – Tuesday, February 24, 1998

NOTE: This is an **OPEN book** exam. This means you **MAY** use any notes, books, pocket calculators or any other learning aids you have brought with you. What you **MAY NOT** use is the brain of people sitting around you. **You are under the University's Honor Code**, and therefore are pledged to hand in work which is entirely your own. Speaking of work, make sure you **show all of your work,** since questions will be graded with the possibility of earning partial credit; in addition, the instructor tends to look incredulously at correct answers without any work showing how they have been obtained. Finally, **hand in all your exam sheets in the blue booklet.**

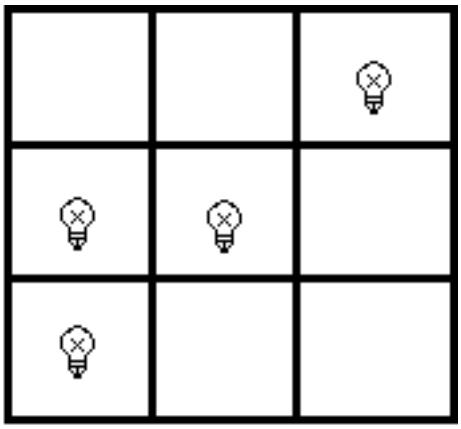
NAME _____

Make sure you have four (4) pages of questions, excluding this cover sheet.
The exam has five (V) questions.

I. *(15 pts.)* This question deals with the original version of the first game studied in class (the very first version we studied.) Shown below is a "START" configuration and a "TARGET" configuration. Tell me which buttons need to be pushed in order to go from the START to the TARGET. No button can be pushed twice.



START



TARGET

ANSWER: _____

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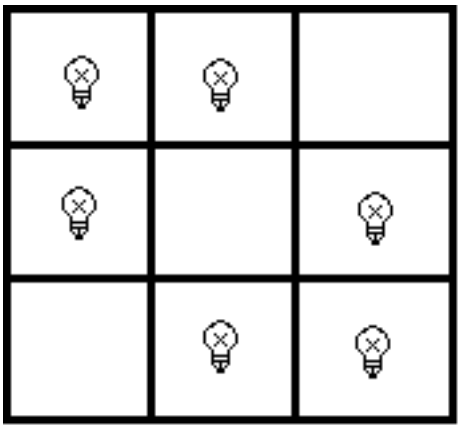
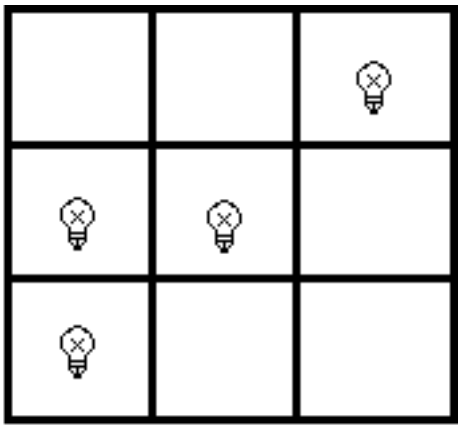


START



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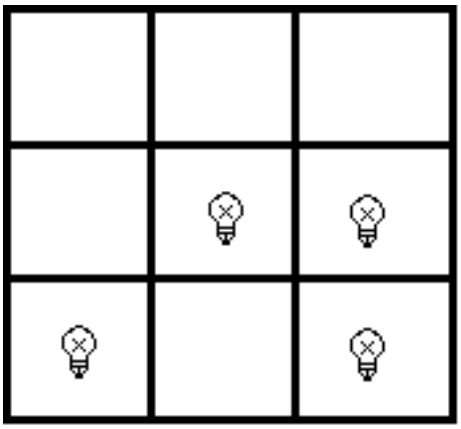
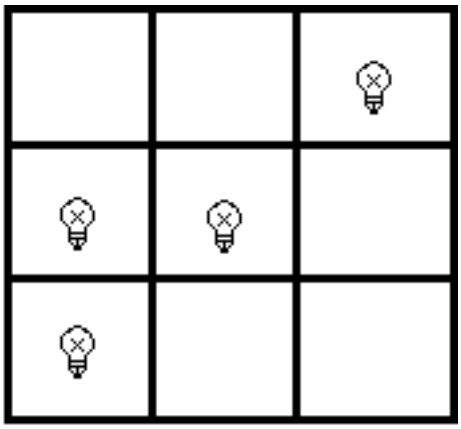


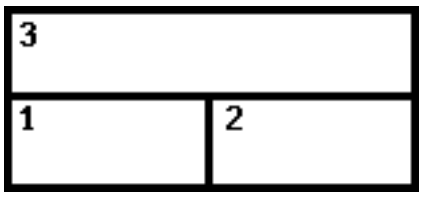
START



TARGET

ANSWER: _____

II. *(30 pts.)* This question deals with the game shown below, where each "button" can have one of three colors, **A**qua *(a pale blue)*, **B**uff *(a brownish yellow)* and **C**rimson *(a fiery red.)* With the buttons numbered as shown the game works as follows:



- The colors on each button change by following the sequence

Aqua —> **B**uff —> **C**rimson —> *(back to)* **A**qua

- The action of each button is as follows:

button no. 1	advances button no. 2 **one step** in the sequence advances button no. 3 **two steps** in the sequence
button no. 2	advances button no. 1 **two steps** in the sequence advances button no. 3 **one step** in the sequence
button no. 3	advances button no. 1 **one step** in the sequence advances button no. 2 **two steps** in the sequence advances button no. 3 **one step** in the sequence

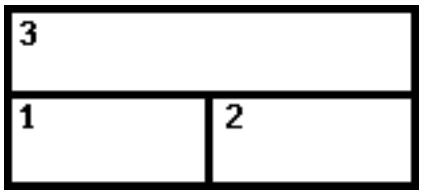
Action Table

(A) *(5 pts.)* What is the modulus of the arithmetic regulating the game?

(B) *(10 pts.)* Set up the system needed to solve the game.

(C) *(15 pts.)* Use the Gauss-Jordan elimination method to obtain the solutions of the game.

III. *(30 pts.)* This question deals with the the game shown below, where each "button" can have one of five colors, **A**qua *(a pale blue)*, **B**uff *(a brownish yellow)* , **C**rimson *(a fiery red.)*, **D**andelion *(a bright yellow)*, and **E**cru *(pale tan.)*



The colors on each button change by following the sequence

Aqua —> **B**uff —> **C**rimson —> **D**andelion —> **E**cru —> *(back to)* **A**qua

(A) *(5 pts.)* What is the modulus of the arithmetic regulating the game?

(B) *(15 pts.)* With the buttons numbered as shown, the system whose solution solves the game is **(do NOT solve the system !)**

$$\begin{cases} 4\ 0\ 3 = \mathbf{1} \\ 1\ 1\ 2 = \mathbf{2} \\ 3\ 1\ 0 = \mathbf{3} \end{cases}$$

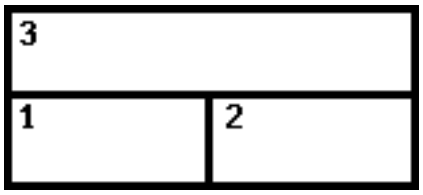
Fill all the blanks in the Action Table below.

button no. 1	advances button no. 1 _____ in the sequence advances button no. 2 _____ in the sequence advances button no. 3 _____ in the sequence
button no. 2	advances button no. 1 _____ in the sequence advances button no. 2 _____ in the sequence advances button no. 3 _____ in the sequence
button no. 3	advances button no. 1 _____ in the sequence advances button no. 2 _____ in the sequence advances button no. 3 _____ in the sequence

Action Table

(C) *(10 pts.)* What are the essential ingredients for the general type of games we have studied in Topic 1 ?

III. *(30 pts.)* This question deals with the the game shown below, where each "button" can have one of five colors, **A**qua *(a pale blue)*, **B**uff *(a brownish yellow)* , **C**rimson *(a fiery red.)*, **D**andelion *(a bright yellow)*, and **E**cru *(pale tan.)*



The colors on each button change by following the sequence

Aqua —> **B**uff —> **C**rimson —> **D**andelion —> **E**cru —> *(back to)* **A**qua

(A) *(5 pts.)* What is the modulus of the arithmetic regulating the game?

(B) *(15 pts.)* With the buttons numbered as shown, the system whose solution solves the game is **(do NOT solve the system !)**

$$\begin{cases} 3\ 1\ 0 = \mathbf{1} \\ 1\ 1\ 2 = \mathbf{2} \\ 4\ 0\ 3 = \mathbf{3} \end{cases}$$

button no. 1	advances button no. 1 _____ in the sequence advances button no. 2 _____ in the sequence advances button no. 3 _____ in the sequence
button no. 2	advances button no. 1 _____ in the sequence advances button no. 2 _____ in the sequence advances button no. 3 _____ in the sequence
button no. 3	advances button no. 1 _____ in the sequence advances button no. 2 _____ in the sequence advances button no. 3 _____ in the sequence

Fill all the blanks in the Action Table below.

Action Table

(C) *(10 pts.)* What are the essential ingredients of the general type of games we have studied in Topic 1 ?

IV. *(15 pts.)* Let $\mathbf{P}$ be the permutation shown below in the two-row notation.

$$\mathbf{P} = \begin{pmatrix} A & B & C & D & E & F & G & H & I & J \\ G & I & F & E & A & B & H & J & C & D \end{pmatrix}$$

(A) *(5 pts.)* Write $\mathbf{P}$ as the product of disjoint cycles.

_____

(B) *(5 pts.)* Write $\mathbf{P}$ as the product of nine transpositions.

_____

(C) *(5 pts.)* Write $\mathbf{P}$ as the product of ten transpositions.

_____

V. *(10 pts.)* Let $\mathbf{P}$ be the permutation shown below in the cycle notation. ***(Note the blank space!)***

$$\mathbf{P} = (\text{BJWEHYTC G})(\text{ADLNOMQZ})(\text{SRIXPFKVU})$$

(A) *(5 pts.)* Encode **<u>your full name</u>**, in the form

FIRST LAST

using the permutation $\mathbf{P}$.

_____

(B) *(5 pts.)* I have used $\mathbf{P}$ to encode a message for you. I got

JSMODGKMICSOD

What message am I sending you?

_____