Math. 103 - Processes of Mathematical Thought
Prof. Mario Borelli
Exam II – Tuesday, March 24, 1998

NOTE: This is an **OPEN book** exam. This means you **MAY** use any notes, books, pocket calculators or any other learning aids you have brought with you. What you **MAY NOT** use is the brain of people sitting around you. **You are under the University's Honor Code**, and therefore are pledged to hand in work which is entirely your own. Speaking of work, make sure you **show all of your work,** since questions will be graded with the possibility of earning partial credit; in addition, the instructor tends to look incredulously at correct answers without any work showing how they have been obtained. Finally, **WRITE ALL OF YOUR ANSWERS AND EXPLANATIONS IN THE BLUE BOOKLET.**

Make sure you have three (3) pages of questions.
The exam has four (IV) questions
and one *extra credit* question.

I. *(20 pts.)* This question deals with the "road toy" we have studied in class. Shown below are two configurations of the last seven tiles. You may assume that in both configurations all of the other 13 tiles are in the desired alphabetical order. For each configuration tell me how many pivots are needed to alphabetize all 20 tiles. Identify pivots precisely as follows:

- if you think you need seventeen consecutive pivots, tell me the first four tiles you pivot.
- if you think you need the eight pivots 31431412 or 21413413 used in class, tell me also which is the zero-th tile.
- if you think you need a single pivot, tell me which four tiles you are pivoting.

Explain your answer clearly!

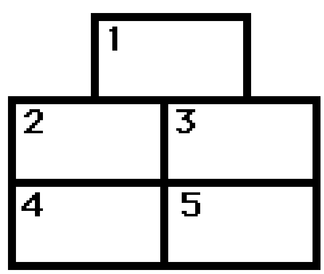
Configuration No. 1 *(15 pts.)*

N	O	P	R	Q	S	T

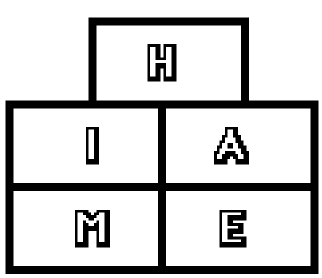
Configuration No. 2 *(15 pts.)*

N	O	R	S	P	Q	T

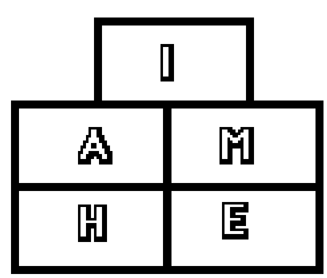
II. *(30 pts.)* This question deals with the toy shown below, consisting of six tiles, numbered as shown for your convenience.



(A) *(15 pts.)* In this version of the game you can perform the following operations:
Rotate 1, 2 and 4, that is, apply the cycle (124) as often as you please.
Rotate 1, 3 and 5, that is, apply the cycle (135) as often as you please.
Rotate 1, 4 and 5, that is, apply the cycle (145) as often as you please.
Your starting configuration is:



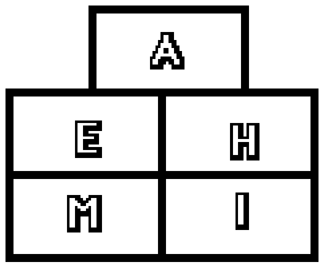
and your target is



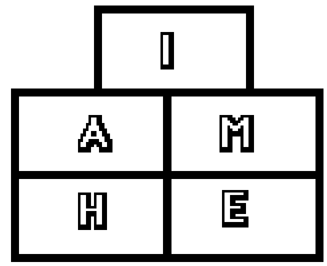
(the target is what Jesus answered when asked in Gehtsemani if He was the Nazorene.
This is Lent!)

If one can reach the target from the start, using ONLY the given rotations, do it.
If one cannot, explain why not.

(B) *(15 pts.)* In this version of the game you can perform the following operations:
Exchange any two tiles which share a common, positive piece of border.
NOTE that you cannot exchange, for example, 1 with 4, nor 2 with 5, but you can exchange, for example, 1 with 2, and 2 with 4.
Your starting configuration is:



and your target is



If one can reach the target from the start, using ONLY the given exchanges, do it. If one cannot, explain why not.

III. *(20 pts.)* Some of the networks described below can be drawn, some cannot. Draw the feasible ones and explain why it is not possible to draw the others.

(A) *(5 pts.)* A one-piece network with six odd vertices.

(B) *(5 pts.)* A one piece network with seven odd vertices..

(C) *(5 pts.)* A non-eulerian one-piece network with three vertices.

(D) *(5 pts.)* A non-eulerian one-piece network with four vertices.

IV. *(30 pts.)* A federal highway inspector based in Washington DC has been given the job of inspecting all the toll roads connecting the six cities shown diagram 1 in the state of Montigan (a mythical state), and then inspect all the toll roads connecting the six cities shown in diagram 2 in the state of Pennsylbama (another mythical state.) She plans to fly from Washington to Montigan, drive the toll roads, then fly to Pennsylbama, drive the toll roads, then fly back to Washington. She plans to travel incognito, which means she will have to pay the tolls like everybody else. Unless otherwise indicated in the diagrams, all tolls between any two cities are $ 2.00 each. Schedule precisely (city by city, starting and ending in Washington) the least expensive itinerary for her. Air travel prices are constant between states, and from and to Washington, DC.

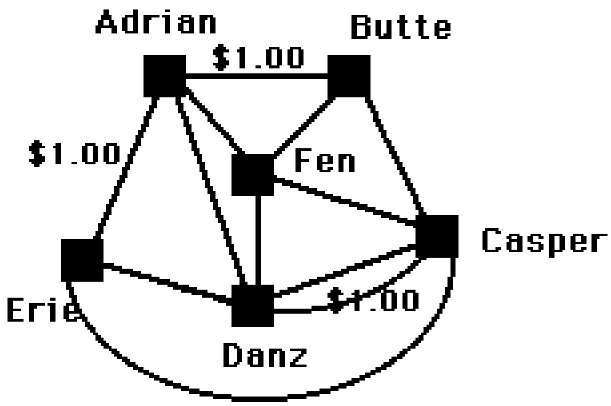


Diagram 1

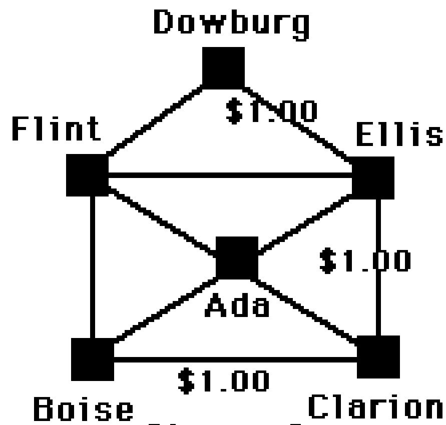


Diagram 2

EXTRA CREDIT. *(15 pts.)*: Show that in version **(B)** of the game of question II, any target is reachable from any start. [HINT: Compute the permutation (XY)(YZ)(XY) and apply judiciously to show that every transposition of any two tiles is feasible.]