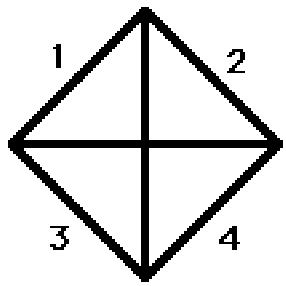
Math. 103 - Processes of Mathematical Thought
Prof. Mario Borelli
Final Examination – Thursday, May 7, 1998

NOTE: This is an **OPEN book** exam. This means you **MAY** use any notes, books, pocket calculators or any other learning aids you have brought with you. What you **MAY NOT** use is the brain of people sitting around you. **You are under the University's Honor Code**, and therefore are pledged to hand in work which is entirely your own. Speaking of work, make sure you **show all of your work,** since questions will be graded with the possibility of earning partial credit; in addition, the instructor tends to look incredulously at correct answers without any work showing how they have been obtained. Finally, **WRITE ALL OF YOUR ANSWERS IN THE BLUE BOOKLET(S)**

Make sure you have four (4) pages of questions,
excluding this cover sheet.
The exam has six (VI) questions.

I. *(25 pts.)* This question deals with the game shown below, where each "button" can have one of three colors, **A**qua *(a pale blue)*, **B**uff *(a brownish yellow)* and **C**rimson *(a fiery red.)* With the buttons numbered as shown the game works as follows:



- The colors on each button change by following the sequence

Aqua —> **B**uff —> **C**rimson —> *(back to)* **A**qua

- The action of each button is as follows:

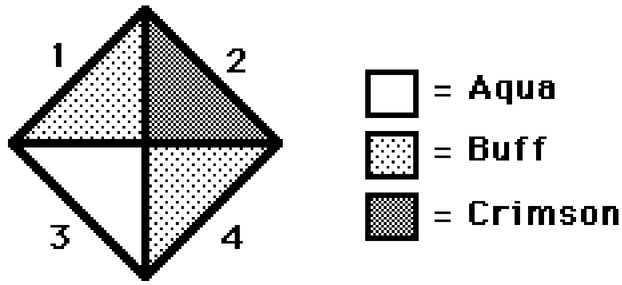
Action Table

button no. 1	advances button no. 1	**two steps** in the sequence
	advances button no. 3	**one step** in the sequence
button no. 2	advances button no. 1	**one steps** in the sequence
	advances button no. 3	**two steps** in the sequence
	advances button no. 4	**one steps** in the sequence
button no. 3	advances button no. 1	**one steps** in the sequence
	advances button no. 2	**one steps** in the sequence
	advances button no. 4	**two step** in the sequence
button no. 4	advances button no. 1	**one step** in the sequence
	advances button no. 4	**one step** in the sequence

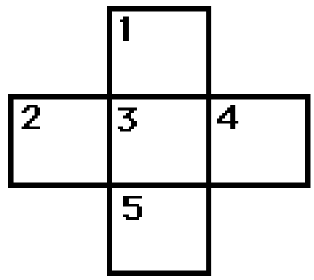
(A) *(5 pts.)* Set up the system needed to solve the game.

(B) *(12 pts.)* Use the Gauss-Jordan elimination method to obtain the solutions of the game.

(C) *(8 pts.)* Apply the solutions you found to determine which buttons are to be pushed **(no button more than twice)** to turn all buttons Crimson, starting from


1
2
3
4
= Aqua
= Buff
= Crimson

II. ***(25 pts.)*** This question deals with the toy shown below, consisting of five tiles, numbered as shown for your convenience.

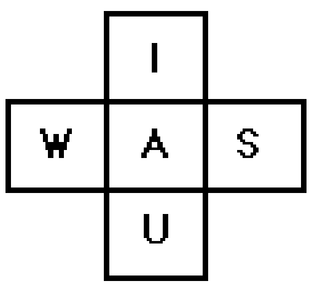


You can perform the following operations:

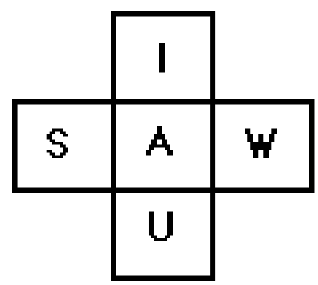
- Rotate any three distinct tiles, provided tile no. 3 is one of them, as often as you please.

(1) *(5 pts.)* Using the numbering shown above, list the six rotations you can perform.

(2) *(10 pts.)* Your starting configuration is:

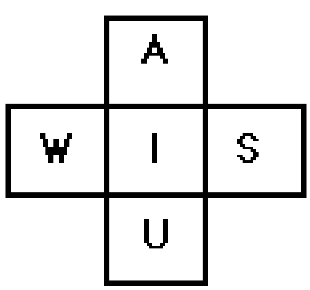


and your target is

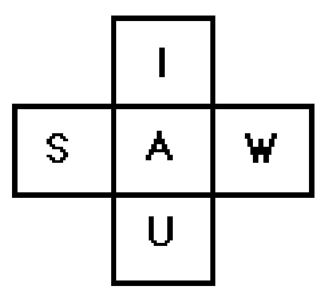


If one can reach the target from the start, using ONLY the given rotations, do it.
If one cannot, explain why not.
(NOTE: You don't need too many computations.)

(3) *(10 pts.)* Your starting configuration is:



and your target is



If one can reach the target from the start, using ONLY the given rotations, do it.
If one cannot, explain why not.
(NOTE: You don't need too many computations.)

III. *(25 pts.)* Let **P** and **Q** be the permutations shown below in the two-row notation.

$$\mathbf{P} = \begin{pmatrix} A & B & C & D & E & F & G & H & I & J \\ G & I & F & E & A & B & H & J & C & D \end{pmatrix}$$

$$\mathbf{Q} = \begin{pmatrix} A & B & C & D & E & F & G & H & I & J \\ H & G & I & F & B & D & A & C & J & E \end{pmatrix}$$

(A) *(6 pts.)* Write **Q** as the product of disjoint cycles.

(B) *(9 pts.)* Write **QPQ** as the product of disjoint cycles.

(C) *(10 pts.)* Determine the parity of **Q, PQP (think!)** and **PQPQ (think!)** respectively**.**

IV. (25 pts.) Let **N** be a **one piece non-Eulerian 1-network** which is known to have exactly five vertices.

(A) *(12 pts.)* Show that one can always **add exactly one edge** to **N** and obtain a one piece Eulerian 1-network **M**.

(B) *(13 pts.)* Show an example in which one CANNOT **delete exactly one edge** from **N** and obtain a one piece Eulerian 1-network **M**.

V. (25 pts.) For each one of the solids defined below, either draw its planar representation or explain why it cannot be done.

(A) *(8 pts.)* The faces of the solid consist exactly of **four** triangles and **four** pentagons.

(B) *(8 pts.)* The solid has exactly 12 vertices, each vertex belongs to exactly three faces, the faces consist only of quadrilaterals and hexagons.

(C) *(8 pts.)* The solid has exactly 10 vertices, each vertex belongs to exactly three faces, the faces consist only of triangles and quadrilaterals.

VI. **(25 pts.)** Here are three ballots on four candidates **A**dams, **B**rummels, **C**ollins and **D**iethers. Decide which two ballots are the closest....:

(A) *(12 pts.)* ...when you compute the distance as the sum of the absolute values of the individual differences.

(B) *(13 pts.)* ...when you compute the distance as the sum of **the squares of** the individual differences.

Candidates	Ballot No. I	Ballot No. II	Ballot No. III
Adams	2	1	4
Brummels	2	2	3
Collins	3	1	2
Diethers	3	2	3

Good luck to all of you!
I have enjoyed this semester with you.
Have a happy, profitable and dsafe summer!