

1. Let U be the set of cars in a certain parking lot. Let
 $A = \{\text{Fords}\}$ $B = \{\text{red cars}\}$ $C = \{\text{hatchbacks (i.e. 3-doors)}\}$
Which of the following is in $(A \cup B) \cap C$? (assume that a car cannot be both red and blue)

- a. a red Honda hatch back
- b. a red Ford hatchback
- c. a red Honda 4-door
- d. a blue Honda 4-door
- e. a blue Honda hatchback

2. Let S and T denote subsets of a universal set U . Assume that
 $n(U) = 100$, $n(S \cup T') = 60$, $n(S \cap T') = 20$, $n(S \cap T) = 10$. Find $n(T)$.

- a. 40
- b. 30
- c. 20
- d. 10
- e. 50

3. A group of 100 college athletes were asked what sports they played in high school. a total of 50 played basketball, 40 played volleyball and 35 played soccer. Furthermore, 10 played soccer but neither volley ball nor basketball, and 10 played all three sports. Finally, a total of 15 played basketball and volleyball, and 25 played volleyball and soccer. How many didn't play any of the three sports?

- a. 0
- b. 15
- c. 5
- d. 10
- e. 20

4. A casting director wants to choose the actors to portray a family consisting of a father, a mother, two boys and two girls. His pool of actors consists of 2 men, 3 women, 4 boys and 2 girls. In how many ways can he choose the actors for the family?

- a. 36 b. 48 c. 72 d. 24 e. 12

5. Eleven college students decide to play a game of basketball. They will choose two teams of five students each, and the last student will be the referee. In how many ways can they do this?

- a. $\binom{11}{5} \binom{6}{5}$ b. $\binom{11}{5} \binom{6}{5} \binom{1}{11}$ c. $\frac{1}{2} \binom{11}{5} \binom{6}{5}$
d. $\frac{11!}{5!}$ e. $2 \binom{11}{5} \binom{6}{5}$

6. A patrol of 10 Boy Scouts is hiking when one is bit by a snake. they decide that at least one of the remaining 9 Scouts should stay with the victim and the rest (but at least one, obviously) should go for help. IN how many ways can they choose the group that will go for help?

- a. $2^9 - 1$ b. 2^9 c. 2^{10} d. $2^9 - 2$ e. $2^{10} - 3$

7. Groucho, Chico, Harpo and Zeppo are the four members of a club. they decide that each of them will be president of the club, each for some 3 of the months of the year, not necessarily consecutive. In how many ways can they decide for which 3 months each of them will be president?

- a. $\frac{1}{4!} \frac{12!}{(3!)^4}$ b. $\frac{12!}{(3!)^4}$ c. $\frac{12!}{(4!)^3}$ d. $\frac{1}{3!} \frac{12!}{(4!)^3}$ e. $\binom{12}{3}^4$

8. Alice, Bill, Graig and Doug each draws a card from a standard deck (without replacement). consider the following events:

A = "Alice draws a red Queen"

B = "Bill draws a red Queen"

C = "Craig draws a red Queen"

D = "Doug draws a red Queen"

(Note that there are two red Queens and two black Queens in a standard deck.) Which of the following pairs of events are mutually exclusive?

- a. $A \cap B$ and C' b. $A \cup B$ and $C \cup D$ c. A and B'
d. $B \cap C$ and D' e. $A \cap B$ and $C \cup D$

9. A red die and a green die are tossed, and the numbers on the uppermost faces are observed. What are the odds that these numbers add up to 7?

- a. 6 to 1 b. 1 to 6 c. 5 to 1 d. 1 to 5 e. 5 to 31

10. Irving has 5 pairs of socks: blue, black, green, red and hot pink. These 10 socks are all mixed together in a drawer, and 3 are selected at random. What is the probability that 2 of the 3 selected socks match?

a. $\frac{5 \cdot 8}{\binom{10}{3}} = \frac{1}{3}$ b. $\frac{3}{10}$ c. $\frac{3}{\binom{10}{3}} = \frac{1}{40}$ d. $\frac{8 \binom{10}{2}}{\binom{10}{3}} = \frac{3}{8}$ e. $\frac{5}{\binom{10}{3}} = \frac{1}{24}$

11. A deck of cards is handed to each of 10 people, in turn. Each person is asked to pick a card at random, look at it, and put it back in the deck. Find the probability that at least two of the people pick the same card.

~~a. $\frac{1 - \frac{C(52,10)}{P(52,10)}}{(52)^{10}}$~~ ~~b. $\frac{P(52,10)}{(52)^{10}}$~~ c. $1 -$

d. $1 - \frac{10}{52}$ ~~e. $\frac{C(52,2)}{P(52,2)}$~~

12. Let A and B be events and suppose that $P(A) = 0.3$, $P(B) = 0.2$, $\Pr(A|B) = 0.5$. Find $\Pr(B|A)$.

a. $\frac{2}{3}$ b. $\frac{1}{3}$ c. $\frac{2}{5}$ d. $\frac{3}{5}$ e. $\frac{3}{4}$

13. At a certain university, 1000 students took introductory French in the first semester. Of these, Prof. Deindonné taught 400, Prof. Serre Taught 250, Prof. Grothendieck taught 200 and Prof. Deligue taught 150. It turned out that a final grade of A was received by 50% of Prof. Diendonné's students, 40% of Prof. Serre's students, 25% f Prof. Grothendieck's students and 20% of Prof. Deligue's students. A student was selected at random among those who took introductory French, and she was found to have received an A. what is the probability that she was in Prof. Delique's class?

- a. $\frac{3}{38}$ b. $\frac{150}{1000}$ c. $\frac{20}{100}$ d. $(.15)(.2)$ e. $\frac{15}{38}$

14. Jake has two urns. Urn I contains 3 white balls and 2 red balls, while Urn II contains 3 white balls and 5 red balls. He rolls a die and observes the number that comes up. If this number is a 1 or a 2 he draws a ball from Urn I. If it is a 3, 4, 5 or 6 he draws a ball from Urn II. What is the probability that he will draw a white ball?

- a. $\frac{9}{40}$ b. $\frac{6}{13}$ c. $\frac{1}{20}$ d. $\frac{9}{20}$ e. $\frac{39}{40}$

15. In a certain carnival game, 3 green balls and 1 red ball are placed in an urn. The player selects one ball at a time without replacement. If the ball is green he receives \$1.00, and if it is red he pays \$2.00. The game ends as soon as he either gets the red ball or has gotten 3 green balls. Find the probability distribution for his earnings.

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16. Bob has a probability of $\frac{1}{2}$ that he will pass his driver's test on any given attempt. His state has a rule that a person can take the exam at most 3 times. Bob plans to keep taking the test until he either passes it or reaches the limit of 3 attempts. What is the expected number of times that he will take the exam?

- a. 1 b. $\frac{3}{2}$ c. 2 d. $\frac{11}{8}$ e. $\frac{7}{4}$

17. The following is the probability distribution for a certain experiment:

outcome	Prob
3	1/3
4	1/3
5	1/3

Find the variance.

- a. 1 b. $\frac{2}{3}$ c. 0 d. $\frac{4}{3}$ e. 2

18. In the Billionaire Bonanza Lottery, seven digits (each one 0 through 9) are chosen at random. Michelle predicts that exactly three of the seven digits will be an 8. What is the probability that she is correct?

- a. $\binom{7}{3} \binom{1}{10}^3 \binom{9}{10}^4$ b. $\frac{3}{7}$ c. $\binom{1}{10}^7$
d. $\binom{7}{3} \binom{1}{10}^7$ e. $\binom{1}{10}^3 \binom{9}{10}^4$

19. Find the area under the standard normal curve to the right of $z = \frac{1}{2}$. (Use the attached table)

- a. .5000 b. .6915 c. - .3085 d. .3085 e. .2500

20. The birthweights of newborns in a certain population is normally distributed with mean 4 kg and standard deviation $\frac{1}{2}$ kg. Mrs. Jones is told that 98% of all newborns weigh more than her baby does. How much does her baby weigh? (Use the attached table.)

- a. 3.92 kg b. 2.05 kg c. 5.025 kgd. 1.025 kg e. 2.975 kg

21. An archer has a probability of $\frac{2}{3}$ that he will hit the bullseye on any given attempt. Suppose he makes 18 attempts. use the normal approximation to the binomial distribution, and the attached table, to estimate the probability that he will hit the bullseye between 10 and 15 times (inclusive).

- a. .9599 b. $\frac{1}{3}$ c. .7745 d. .8543 e. .9936

22. Find the y-intercept of the line through the point (3,-2) and parallel to the line $2x + 3y = 4$

- a. 0 b. -2 c. 2 d. $\frac{-2}{3}$ e. $\frac{4}{3}$

23. The matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 4 \\ 0 & 3 & 7 \end{bmatrix}$ is pivoted about the circled entry. Find the entry in the 3rd row and 3rd column of the resulting matrix.

- a. 0 b. 1 c. -2 d. 3 e. 7

24. For a certain system of equations, the Gauss-Jordan elimination method yields the following (i.e. the reduced row-echelon form:

$$\begin{array}{cccc|c} x & y & z & w & \\ \hline 1 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

Which of the following is the general solution to this system of equations?

- | | | |
|--|---|---|
| <p>a. $x = 0$
 $y = 0$
 $z = 0$
 $w = 0$</p> | <p>b. $x = 2 - y - w$
 $y = 2 - w$
 $z = 3 - w$
 $w = 0$</p> | <p>c. $x = 2 - y - w$
 $y = 2 - w$
 $z = 3 - w$
 $w = \text{any value}$</p> |
| <p>d. $x = 2 - y - w$
 $y = \text{any value}$
 $z = 3 - w$
 $w = \text{any value}$</p> | <p>e. $x = 2 - y - w$
 $y = \text{any value}$
 $z = 3 - w$
 $w = 0$</p> | |

25. Let $A = \begin{bmatrix} -2 & -2 \\ 4 & 3 \end{bmatrix}$ and let A^{-1} be its inverse. Find the entry in the second row and first column of A^{-1} .

- a. -1 b. 2 c. -2 d. 1 e. 4

26. When you multiply the matrices

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 & -2 \\ -1 & 1 & 3 \\ 2 & -1 & 2 \end{bmatrix}$$

the entry in the first row and second column of the product is

- a. these matrices cannot be multiplied b. 9 c. 4
d. 10 e. 1

27. When the following system of linear equations is solved, what is the value of z ?

$$\begin{aligned} x &+ 2z = 8 \\ 2x + y &= 3 \\ 2x + 2y + 5z &= 17 \end{aligned}$$

- a. 1 b. 2 c. 3 d. 4 e. 5

28. Maximize the objective function $2x + 5y$ subject to the constraints

$$\begin{aligned}2x + y &\leq 10 \\x + 2y &\leq 8 \\x &\geq 0 \\y &\geq 0\end{aligned}$$

The maximum value is

- a. 50 b. 18 c. 10 d. 50 e. 16

29. Set up the following linear programming problem:

A cafeteria plans to make veggie pizza for tonight's dinner. They will make two sizes, small and large. A small pizza requires 3 oz of sauce and 8 oz of topping, and brings them a profit of \$1.00. A large pizza requires 7 oz of sauce and 12 oz of topping and brings a profit of \$2.00. They have 1000 oz of sauce and 2000 oz of topping available. How many of each size pizza should they make in order to maximize profit?

(Let $x = \#$ small, $y = \#$ large.)

a. Maximize $x + 2y$
subject to
 $3x + 7y \leq 1000$
 $8x + 12y \leq 2000$
 $x \geq 0$
 $y \geq 0$

b. Maximize $x + 2y$
subject to
 $3x + 8y \leq 1000$
 $7x + 12y \leq 2000$
 $x > 0$
 $y \geq 0$

c. Maximize $x + 2y$
subject to
 $7x + 3y \leq 1000$
 $12x + 8y \leq 2000$
 $x \geq 0$
 $y \geq 0$

d. Maximize $x + 2y$
subject to
 $3x + 7y \geq 1000$
 $8x + 12y \geq 2000$
 $x \geq 0$
 $y \geq 0$

e. Maximize $x + 2y$
subject to
 $3x + 8y \geq 1000$
 $7x + 12y \geq 2000$
 $x > 0$
 $y \geq 0$

30. Set up the following linear programming problem.

Mr. Smith plans to invest \$10,000 in three types of stocks: low-risk, medium-risk and high-risk. He has the following three guidelines for investment:

1. At least half the money must be invested in low-and medium-risk stocks.
2. The amount invested in high-risk stocks must be at least \$1000 more than the amount invested in low-risk stocks.
3. No more than \$6000 can be invested in medium and high-risk stocks (combined). The expected yields are 7% for low-risk, 10% for medium-risk and 12% for high risk stocks. How much money should he invest in each type of stock in order to maximize total expected yield? (Let x = amount invested in low-risk, y = amount invested in medium-risk)

<p>a. Maximize $12(.05)x - (.02)y$</p> <p>subject to $x + y \geq 5000$</p> <p style="padding-left: 2em;">$x - y \geq 1000$</p> <p style="padding-left: 2em;">$x + y \geq 6000$</p> <p style="padding-left: 4em;">$x \geq 0$</p> <p style="padding-left: 4em;">$y \geq 0$</p>	<p>b. Maximize $1200 - (.05)x - (.02)y$</p> <p>subject to $x + y \geq 5000$</p> <p style="padding-left: 2em;">$2x + y \leq 9000$</p> <p style="padding-left: 4em;">$x \geq 4000$</p> <p style="padding-left: 4em;">$x \geq 0$</p> <p style="padding-left: 2em;">$x + y \leq 10,000$</p>
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<p>a. Maximize $1200 - (.05)x - (.02)y$</p> <p>subject to $x + y \leq 5000$</p> <p style="padding-left: 2em;">$3x - 4y \leq 6000$</p> <p style="padding-left: 4em;">$x \leq 4000$</p> <p style="padding-left: 4em;">$x \geq 0$</p> <p style="padding-left: 4em;">$y \geq 0$</p> <p style="padding-left: 2em;">$x + y \geq 10,000$</p>	<p>b. Maximize $1200 - (.05)x - (.02)y$</p> <p>subject to $x + y \geq 5000$</p> <p style="padding-left: 2em;">$3x + 2y \leq 8000$</p> <p style="padding-left: 4em;">$x \leq 2000$</p> <p style="padding-left: 4em;">$x \geq 0$</p> <p style="padding-left: 4em;">$y \geq 0$</p>
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a. Maximize $(.07)x + (.10)y$

subject to $x + y \geq 5000$

$3x + y \leq 8000$

$x + y \leq 9000$

$x \geq 0$

$y \geq 0$

