Formulas: $F = (1 + i)^n P$ $F = s_{n\,i} R$ $P = a_{n\,i} R$

$$s_{n\,i} = \frac{(1+i)^n - 1}{i} \qquad a_{n\,i} = \frac{(1+i)^n - 1}{i(1+i)^n}$$

If $A = \begin{bmatrix} I & S \\ 0 & R \end{bmatrix}$ is an absorbing stochastic matrix then the stable matrix of A is $\begin{bmatrix} I & S(I - R)^{-1} \\ 0 & 0 \end{bmatrix}$. [*Note* : The identity matrix I in $(I - R)^{-1}$ is chosen to be the same size as R in order to make the matrix subtraction permissible.]

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1. Consider the matrix $A = \begin{bmatrix} 0.2 & 0.2 & 0.5 \\ 0.3 & 0.3 & 0.3 \\ 0.1 & 0.5 & 0.2 \end{bmatrix}$. Which of the following statements about A is true?

a. A is stochastic.
b. A is stochastic and regular.
c. A is stochastic and absorbing.
d. A is stochastic, regular but not absorbing.
e. A is not stochastic.

2. Consider the stochastic matrix $A = \begin{bmatrix} 1 & 0.2 \\ 0 & 0.8 \end{bmatrix}$. Then A is:

a. absorbing and regular
b. absorbing but not regular
c. regular but not absorbing
d. stable but not absorbing
e. neither absorbing nor regular

3. Let $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ be the transition matrix of a Markov process. If the distribution matrix of the current generation is $\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$. Then the distribution of the next generation is:

a. $\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$ b. $\begin{bmatrix} \frac{2}{9} \\ \frac{4}{9} \\ \frac{4}{9} \end{bmatrix}$ c. $\begin{bmatrix} \frac{3}{9} \\ \frac{2}{9} \\ \frac{4}{9} \end{bmatrix}$ d. $\begin{bmatrix} \frac{2}{9} \\ \frac{3}{9} \\ \frac{4}{9} \end{bmatrix}$ e. $\begin{bmatrix} \frac{1}{9} \\ \frac{2}{9} \\ \frac{4}{9} \end{bmatrix}$

4. A Markov process with three states has transition matrix

	Current State		
	I	II	III
Next I	0.2	0.3	0.4
State II	0.1	0.2	0
III	0.7	0.5	0.6

The probability of proceeding from (current) state III to (next) state I is:

a. 0.2 b. 0.3 c. 0.4 d. 0.5 e. 0.7

5. The stable distribution $\begin{bmatrix} x \\ y \end{bmatrix}$ of the matrix A = $\begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}$ is:

a. $\begin{bmatrix} \frac{6}{11} \\ \frac{5}{11} \end{bmatrix}$ b. $\begin{bmatrix} \frac{7}{11} \\ \frac{4}{11} \end{bmatrix}$ c. $\begin{bmatrix} \frac{4}{9} \\ \frac{5}{9} \end{bmatrix}$ d. $\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$ e. $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

6. The stable matrix of the transition matrix A = $\begin{bmatrix} 1 & 0 & 0.4 \\ 0 & 1 & 0.5 \\ 0 & 0 & 0.1 \end{bmatrix}$ is

a. $\begin{bmatrix} 1 & 0 & 0.36 \\ 0 & 1 & 0.45 \\ 0 & 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & \frac{4}{9} \\ 0 & 1 & \frac{5}{9} \\ 0 & 0 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 0 & 0 & \frac{4}{9} \\ 0 & 0 & \frac{5}{9} \\ 0 & 0 & 0 \end{bmatrix}$

e. $\begin{bmatrix} 1 & 0 & \frac{5}{9} \\ 0 & 1 & \frac{4}{9} \\ 0 & 0 & 0 \end{bmatrix}$ e. $\begin{bmatrix} 1 & 0 & \frac{1}{10} \\ 0 & 1 & \frac{9}{10} \\ 0 & 0 & 0 \end{bmatrix}$

7. A professor's exams are either easy or hard. If the exam was easy last time, it will be easy this time with a 40% probability. If it was hard last time, it will be easy this time with a 70% probability. The matrix of this Markov process is given by:

a. $\begin{array}{c c} & \text{current} \\ & \begin{array}{cc} \text{easy} & \text{hard} \end{array} \\ \begin{array}{c} \text{easy} \\ \text{hard} \end{array} & \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix} \end{array}$ b. $\begin{array}{c c} & \text{current} \\ & \begin{array}{cc} \text{easy} & \text{hard} \end{array} \\ \begin{array}{c} \text{easy} \\ \text{hard} \end{array} & \begin{bmatrix} 0.3 & 0.4 \\ 0.7 & 0.6 \end{bmatrix} \end{array}$ c. $\begin{array}{c c} & \text{current} \\ & \begin{array}{cc} \text{easy} & \text{hard} \end{array} \\ \begin{array}{c} \text{easy} \\ \text{hard} \end{array} & \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} \end{array}$

d. current
$$\begin{array}{cc} & \begin{array}{cc} \text{easy} & \text{hard} \end{array} \\ \begin{array}{c} \text{easy} \\ \text{hard} \end{array} & \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} \end{array}$$

e. current
$$\begin{array}{cc} & \begin{array}{cc} \text{easy} & \text{hard} \end{array} \\ \begin{array}{c} \text{easy} \\ \text{hard} \end{array} & \begin{bmatrix} 0.4 & 0.7 \\ 0.6 & 0.3 \end{bmatrix} \end{array}$$

8. Which of the matrices are regular:

$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A_3 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ A_4 $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$

a. A_1 b. A_2 c. A_3 d. A_4 e. A_3 and A_4

9. How much money must be deposited now in order to have $10,000 after 10 years if interest is paid at a 6% annual rate compounded monthly?

a. $928.92 b. $5,496.33 c. $3,029.95

d. $7,413.72 e. $5,904.50

10. One thousand dollars is deposited in an account at 8% annual rate compounded quarterly for 2 years. The amount of interest earned during that time is

a. $60.30 b. $90.49 c. $171.66

d. $214.98 e. $306.22

11. How much should Mary save each month to have $20,000 for the down payment to buy a house in 5 years if annual interest rate is 6% compounded monthly?

a. $692.15 b. $511.64 c. $399.68

d. $286.66 e. $201.57

12. If the annual interest rate is 12% compounded daily, the rate per period i is:

a. 1% b. $\frac{12}{365}$ % c. $\frac{1}{2}$ % d. 3% e. 12%

13. How much should Jim deposit in an account paying 12% annual rate compounded monthly so that his son can withdraw $100 at the end of each month for 9 months?

a. $900.00 b. $890.22 c. $856.60

d. $1072.36 e. $998.53

14. If you deposit $10,000 into an account paying 8% annual interest compounded quarterly. How much can you withdraw at the end of each quarter year for 5 years so that balance is zero at the end of 5 years?

a. $611.57 b. $6,115.67 c. $523.72

d. $5,237.23 e. $557.66

15. What is the monthly payment on a 30 year $100,000 mortgage at 12% annual rate compounded monthly?

a. $1543.31 b. $1507.08 c. $1101.08

d. $1,053.22 e. $1,028.61

16. Dan took a loan to buy a car. If the interest rate is 18% compounded monthly and the monthly payment is $200 for 4 years. What is the amount of the loan?

a. $6,808.51 b. $7,808.51 c. $5,532.14

d. $6,532.14 e. $9,172.44

17. Janet took out a loan in the amount of $600. If the annual interest rate is 18% compounded monthly. How much interest did Janet pay at the end of the first month?

a. $108 b. $10.8 c. $1.08 d. $90 e. $9

18. Sam deposits $1,000 per month for 3 years at 12% annual interest rate compounded monthly. How much will Sam have at the end of 3 years?

a. $39,336.10 b. $43,076.88 c. $36,000

d. $36,360 e. $47,275.96

19. If $A = \begin{bmatrix} 0.4 & 1 \\ 0.6 & 0 \end{bmatrix}$ then $A^2 =$

a. $\begin{bmatrix} 0.22 & 0.4 \\ 0 & 0.6 \end{bmatrix}$ b. $\begin{bmatrix} 0.16 & 1 \\ 0.36 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 1.4 & 1 \\ 0.6 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0.76 & 0.4 \\ 0.24 & 0.6 \end{bmatrix}$ e. $\begin{bmatrix} 0.8 & 2 \\ 1.2 & 0 \end{bmatrix}$

20. $1 - 3 + 3^2 - 3^3 + 3^4 - \ldots + 3^{98}$

a. $3^{99} - 1$ b. $\dfrac{3^{99} - 1}{3}$ c. $\dfrac{(-3)^{99} - 1}{-4}$

d. $(-3)^{99}$ e. 3^{99}