1. In a group of people, 35 know how to snow ski, and 25 know how to water ski. If 30 know either how to snow ski or water ski but not both, how many people in the group can both snow ski and water ski?

a. 10 b. 20 c. 15 d. 5 e. 25

2. If A and B are subsets of a universal set U and $A' \cup B = U$, which of the following **must** be true?

a. B is a subset of A b. A is a subset of B c. $A = \varnothing$

d. $B = U$ e. $A \cap B = \varnothing$

3. Let A, B, C be three events in a sample space S such that

$Pr(A) = .28$ $Pr(B) = .51$ $Pr(C) = .50$
$Pr(A \cap B) = .14$ $Pr(B \cap C)\ .19$ $Pr(A \cap C) = .10$
and $Pr(A \cap B \cap C) = .04$?

What is $Pr(A \cup B \cup C)$?

a. .83 b. .84 c. .81 d. .87 e. .90

4. What is the coefficient of $x^3 y^5$ in $(x + y)^8$?

a. 56 b. 54 c. 45 d. 84 e. 48

5. Five children are to be lined up in a row for a photograph. In how many ways can this be done if two of the children (Mary and John) refuse to stand next to each other?

a. 60 b. 72 c. 120 d. 48 e. 65

6. A chemistry lab class containing 20 students is to split into 10 groups of 2 for an experiment. In how many ways can this be done?

a. $\frac{20!}{2^{20} \cdot 10!}$ b. $\frac{20!}{2^{10}}$ c. $\frac{20!}{(10!)^2}$ d. $\frac{20!}{2^{10} \cdot 10!}$ e. $\frac{20!}{2^{10} \cdot (10!)^2}$

7. An urn contains 2 red balls, 2 white balls and 2 green balls. Three balls are drawn without replacement from the urn. What is the probability that the balls are all of different colors?

a. $\frac{2}{3}$ b. $\frac{1}{5}$ c. $\frac{1}{3}$ d. $\frac{1}{2}$ e. $\frac{2}{5}$

8. A fair die is rolled three times. What are the **odds in favor** of obtaining different numbers on the top face in all three rolls?

a. 4:1 b. 4:5 c. 5:4 d. 9:4 e. 4:9

9. What is the probability that a hand of five cards dealt from a standard deck of 52 will contain four cards of one **denomination** (i.e. face value) and one other card?

a. $\frac{13 \cdot 48}{C(52,5)}$ b. $\frac{C(13,4) \cdot 39}{C(52,5)}$ c. $\frac{13 \cdot 48}{P(52,5)}$ d. $\frac{P(13,4) \cdot 39}{P(52,5)}$
e. $\frac{13 \cdot 4 \cdot 48}{C(52,5)}$

10. A carton of 12 eggs on a supermarket shelf contains two eggs with cracks on the bottom, not visible unless they are removed. To check for broken eggs, a shopper removes three randomly chosen eggs from the carton and examines them. What is the probability the customer will find at least one of the broken eggs in the carton this way?

a. $\frac{2}{11}$ b. $\frac{5}{11}$ c. $\frac{1}{6}$ d. $\frac{2}{7}$ e. $\frac{3}{8}$

11. A wealthy student living near South Beach, Miami owns a Rolls Royce. There is a probability of 0.6 that she will go to the beach on a particular day, and of 0.8 that she will drive the Rolls on that day. If she does go to the beach that day, there is a probability of 0.9 that she will drive the Rolls that day. Given that she drove the Rolls on that day, what is the probability that she went to the beach that day?

a. insufficient information to decide. b. $\frac{.6}{.8 \infty .9}$ c. $\frac{.8 \infty .6}{.9}$

d. $\frac{.6 \infty .9}{.8}$ e. $\frac{.6 \infty .9}{.8^2}$

12. A fair coin is tossed four times. Find the probability of obtaining four heads, given that at least three heads were obtained.

a. $\frac{1}{7}$ b. $\frac{1}{16}$ c. $\frac{5}{16}$ d. $\frac{1}{4}$ e. $\frac{1}{5}$

13. An urn contains two white balls, a red ball and a green ball. Balls are drawn one at a time from the urn without replacement, till none are left. Find the probability that the two white balls are drawn before the red ball.

a. $\frac{1}{6}$ b. $\frac{5}{24}$ c. $\frac{1}{3}$ d. $\frac{1}{4}$ e. $\frac{7}{24}$

14. An electrical circuit contains 10 components, which function independently of one another. The probability that a component will fail within 5 years is 0.01. For the circuit to function correctly, all 10 components must work. What is the probability that the circuit will function correctly for five years?

a. $1 - (0.01)^{10}$ b. $(.99)^{10}$ c. $1 - (.99)^{10}$ d. $(.01)^{10}$ e. .1

15. A screw manufacturer has plants in Pittsburgh, Detroit and Chicago. 30% of its screws are manufactured in Pittsburgh, and 1% of those are defective. 40% are manufactured in Detroit, and 2% of those are defective. The remainder of the screws are made in Chicago, and 0.5% of these are defective. If a screw is defective, what is the probability it was made in Pittsburgh.

a. $\frac{6}{25}$ b. $\frac{2}{13}$ c. $\frac{3}{13}$ d. $\frac{1}{10}$ e. $\frac{1}{5}$

16. The following table is the probability distribution table of a random variable X.

k	Pr(X = k)
0	.1
1	.2
2	.3
3	a
4	b

Which of the following is a possible pair of values for a and b compatible with this information?

a. a = .4, b = .5

b. a = .1, b = .3

c. a = .2, b = .3

d. a = .1, b = .5

e. a = .1, b = .2

17. At a mini-golf course, the probability distribution for the number of shots X required on the opening hole is a follows

k	Pr(X = k)
2	$\frac{2}{7}$
3	$\frac{3}{7}$
4	$\frac{2}{7}$

What is the variance of X?

a. $\sqrt{\frac{3}{7}}$

b. $\sqrt{\frac{4}{7}}$

c. $\frac{3}{7}$

d. $\frac{4}{7}$

e. $\sqrt{\frac{2}{7}}$

18. Tom and Mary play a game as follows. A die is rolled; if it shows 6 on the top face, Mary gives Tom $6, otherwise Tom gives Mary $1. What would be the expected value of Tom's winnings if the game were to be played 12 times?

a. $0 b. $1 c. $2 d. – $1 e. – $2

19. The lengths of newly hatched alligators are normally distributed with a mean of 9cm and a standard deviation of 1cm. What percentage of the time a newly hatched alligator will be less than 7.5 cm long?

a. 6.7% b. 8.5% c. 4.2% d. 2.4% e. 1.0%

20. A single fair die is rolled 180 times. Use the normal approximation to the binomial distribution to estimate what percentage of the time the top face shows a six 35 or more times.

a. 22% b. 16% c. 20% d. 18% e. 24%

21. A multiple choice test has 25 questions, each with 5 possible answers. A student is able to answer 15 of these questions correctly, but must guess the remaining answers. What is the probability the student will end up with 20 correct answers out of the 25?

a. $C(15,10)\left(\frac{1}{5}\right)^{5}\left(\frac{4}{5}\right)^{10}$ b. $C(25,20)\left(\frac{1}{5}\right)^{20}\left(\frac{4}{5}\right)^{5}$ c. $C(25,20)\left(\frac{1}{5}\right)^{5}\left(\frac{4}{5}\right)^{20}$

d. $C(15,10)\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^{5}$ e. $C(10,5)\left(\frac{1}{5}\right)^{5}\left(\frac{4}{5}\right)^{5}$

22. At a certain fossil site in Montana, fossils dated at 60 million years are found at a depth of 115' from the top of an exposed cliff face, and fossils dated at 85 million years are found at a depth of 175'. Assuming that fossil age and depth are related by a linear (straight line) equation, at what depth should a paleontologist search for fossils from the time of extinction of the dinosaurs (65 million years ago)?

a. 127' b. 135' c. 142' d. 145' e. 120'

23. Which of the following is the entry in the second row and third column of the matrix C, where $C = \begin{bmatrix} 1 & -4 & 5 \\ -2 & 1 & 3 \\ 6 & -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & -3 \\ -3 & -1 & 5 \\ 5 & 0 & -2 \end{bmatrix}$

a. −1 b. 17 c. 5 d. 27 e. 21

24. Consider the system of equations

$$\begin{cases} 2x + 4y + 7z + 9w = 4 \\ x + 2y + 3z + 4w = 1 \\ 3x + 6y + 4z + 7w = -7 \end{cases}.$$

Which of the following is the correct general solution of this system of equations?

a. $\begin{cases} x = -3 - w \\ y = -1 \\ z = 2 - w \\ w = \text{any number} \end{cases}$

b. $\begin{cases} x = -5 - 2y - w \\ y = \text{any number} \\ z = 2 - w \\ w = \text{any number} \end{cases}$

c. $\begin{cases} x = -5 - 2y - w + 3z \\ y = \text{any number} \\ z = \text{any number} \\ w = \text{any number} \end{cases}$

d. $\begin{cases} x = -5 + 2y - 2w \\ y = \text{any number} \\ z = 2 + w \\ w = \text{any number} \end{cases}$

e. $\begin{cases} x = -3 - 2w \\ y = -1 \\ z = 2 - w \\ w = \text{any number} \end{cases}$

25. Which of the following is the solution for x, y of the matrix equation

$$\begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}.$$

a. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 & -2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

b. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

c. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

d. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

e. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

26. Which of the following is the entry in the second row and third column of the matrix A, where

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 5 & -2 \\ 0 & 0 & 2 \end{bmatrix}^{-1}.$$

a. 7 b. 0 c. −5 d. 5 e. −7

27. Consider the following matrices

$$X = \begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix} \quad Y = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{6} & \frac{3}{4} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} \quad Z = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}$$

Which of these matrices are regular stochastic matrices?

a. X only   b. Y only   c. Z only

d. X and Y only   e. X and Z only

28. A penguin has two ways of spending a day, either on shore in its rookery or at sea, feeding on fish. If the penguin stays in the rookery one day, there is a 90% chance it will go to sea the following day, whereas if it feeds one day, there's only a 70% chance it will feed the next day also. In the long run, what percentage of days does the penguin spend feeding at sea?

a. 85%   b. 80%   c. 75%   d. 83%   e. 78%

29. Solve the following linear programming problem. Find the minimum value of 2x + 3y subject to

$$\begin{cases} x \geq 5 \\ y \geq 5 \\ x + y \geq 20 \\ 2x + y \leq 35 \end{cases}$$

a. 55   b. 40   c. 25   d. 50   e. 45

30. **Set up the following linear programming problem.**

A chemical plant produces two chemicals X and Y. For each gallon of X produced, the plant emits 3 cubic feet of carbon monoxide and 6 cubic feet of sulfur dioxide into the atmosphere. Far each gallon of Y, the plant emits 5 cubic feet of carbon monoxide and 4 cubic feet of sulfur dioxide. Government pollution standards limit the plant to emission of 4000 cubic feet of carbon monoxide and 5600 cubic feet of sulfur dioxide a week. Each gallon of X produced gives a profit of \$2 and each gallon of Y yields a profit of \$1.50. Determine the number of gallons of X and Y to be produced a week to maximize profit. (Let x = number of gallons of X

y = number of gallons of Y).

a. Maximize 2x + 1.5y subject to

$$\begin{cases} 3x + 5y \le 4000 \\ 6x + 4y \le 5600 \end{cases}$$

b. Maximize 1.5x + 2y subject to

$$\begin{cases} 3x + 6y \le 4000 \\ 5x + 4y \le 5600 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

c. Maximize 1.5x + 2y subject to

$$\begin{cases} 3x + 6y \le 5600 \\ 5x + 4y \le 4000 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

d. Maximize 2x + 1.5y subject to

$$\begin{cases} 3x + 5y \le 4000 \\ 6x + 4y \le 5600 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

e. Maximize 2x + 1.5y subject to

$$\begin{cases} 3x + 6y \le 40000 \\ 5x + 4y \le 5600 \end{cases}$$