1. Two subsets A and B of a universal subset U are such that $A' \cap B'$ is the empty set. Which of the following statements is necessarily true? a. A is a subset of B b. $A \cup B = U$ c. $A \cap B$ is empty d. B is a subset of Ae. $A \cap B'$ is empty

2. Tom and Sarah are the two candidates in an election. Each of the 100 voters has the option of ranking the two candidates, just voting for one candidate, or not voting for either of the two candidates. Tom got 50 votes in all, including first and second place votes. Sarah got 60 votes in all and 10 voters did not vote for either of the two candidates. If 5 voters gave Tom the first place vote and Sarah the second place vote, how many voters ranked Sarah # 1 and Tom #2? a. 20 b. 60 c. 30 d. 15 e. 70

3. A coach is asked to rank five teams 1 through 5. If he decides that team A should be ranked above team B, how many choices of different rankings does he have? a. 120 b. 60 c. 56 d. 72 e. 62

4. A hand of seven cards is chosen from a standard deck of 52 cards. How many such hands have 2 cards form three suits each and one card from a fourth suit?

a. $C(4,3) \cdot C(13,2)^3 \cdot 13$ b. $4 \cdot 3 \cdot 2 \cdot C(13,2)^3 C(13,1)$ c. $C(52,2) \cdot C(50,2) \cdot C(48,2) \cdot 46$

d. $C(13,2)^3 \cdot C(13,1)$ e. $52 \cdot C(51,2) \cdot C(49,2) \cdot C(47,2)$

5. A study group has 20 students. In how many different ways can they form four subgroups of 5 each? a. $C(20,5) \cdot C(15,5) \cdot C(10,5)$ b. $\frac{20!}{(5!)^4}$ c. $\frac{20!}{(5!)^4 \cdot 4!}$ d. $\frac{20!}{(4!)^5 \cdot 5!}$ e. $\frac{20!}{(4!)^5}$

6. An urn contains 3 red and 5 green balls. Three balls are drawn <u>without</u> <u>replacement</u>. What is the probability that the three balls have the same color?

a. $\frac{10}{56}$ b. $\frac{11}{56}$ c. $\frac{13}{56}$ d. $\frac{15}{56}$ e. $\frac{21}{56}$

7. An urn contains 3 red and 5 green balls. Three balls are drawn <u>with</u> <u>replacement</u>. What is the probability that the three balls have the same color?

a. $\frac{3!\,P(5,3)}{P(8,3)}$ b. $\left(\frac{3}{8}\right)^3 \cdot \left(\frac{5}{8}\right)^3$ c. $\frac{3 \cdot 5}{P(8,3)}$ d. $\left(\frac{3}{8}\right)^3 + \left(\frac{5}{8}\right)^3$ e. $\frac{3!\,P(5,3)}{C(8,3)}$

8. A partial deck of cards contains 4 red and 3 black cards. Two cards are drawn form this deck, if they are both of the same color you retain these cards - if not, they are replaced and two cards are chosen again (after reshuffling the deck). You keep the cards you get the second time around. Find the probability that the two cards with which you end up are of the same color. a. $\frac{36}{49}$ b. $\frac{40}{49}$ c. $\frac{3}{7}$ d. $\frac{4}{7}$ e. $\frac{33}{49}$

9. A number is chosen at random between 1 and 10, inclusive. If the number chosen is between 1 and 4, inclusive, then a coin is tossed 4 times and the sequence of heads and tails observed. If the number is 5 or higher, then a coin is tossed 5 times and the sequence observed. Given that the sequence consisted of all heads, find the probability that the number chosen was less than 5. a. $\frac{1}{40}$ b. $\frac{4}{7}$ c. $\frac{7}{40}$ d. $\frac{1}{7}$ e. $\frac{3}{7}$

10. An automobile manufacturer has three different suppliers for gaskets. The defect rate of supplier I is 0.5%, that of supplier II is 0.4% and of supplier III is 0.2%. The manufacturer gets 20% of its supply from supplier I, 30% from supplier II and 50% from supplier III. If a randomly chosen gasket is found to be defective, what is the probability that it came from supplier I? a. $\frac{5}{14}$ b. 0.001 c. $\frac{5}{16}$ d. $\frac{5}{11}$ e. .25

11. Given that E and F are independent events and Pr(E) = 0.4, and Pr(F) = 0.5, find Pr(E′ | F). a. 0.2 b. 0.4 c. 0.3 d. 0.6 e. 0.5

12. A company has two divisions I and II. To produce $1 worth of output, Division I requires $0.1 from Division I and $0.4 from Division II. To produce $1 worth of output, division II requires $0.3 from Division I and $0.2 from Division II. The company has an external demand of $30 from Division I and $60 from Division II. How many dollars worth of output should Division I produce to meet this demand?

a. 70 b. 110 c. 80 d. 105 e. 55

13. Mike makes two free throw attempts everyday. He has a 60% chance of making the first free throw. On his second attempt, he has a 50% chance of making it if he missed his first shot and a 70% chance of making his second free throw if he made his first shot. The results of his attempts on one day are independent of the previous days results. Find the probability that he made at least one free throw in the last two days.

a. 0.8 b. 0.91 c. 0.96 d. .6 e. 0.84

14. At a carnival game a card is drawn form a partial deck containing the 4 queens and the ace of spades, without replacement, until a spade is drawn. A player pays $1.50 dollars to play and receives 50 cents for every card drawn. What are the expected earnings of the player in dollars? a. $\frac{1}{2} \cdot \left(\frac{5}{2}\right)\left(\frac{2}{5}\right)^2\left(\frac{1}{5}\right)^3$ b. $-\frac{52}{5}$ c. $\frac{1}{2}$ d. $-\frac{1}{2}$ e. $-\frac{21}{40}$

15. A random variable X has the following probability distribution

k	Pr(X = k
– 12	$\frac{1}{6}$
0	$\frac{1}{3}$
2	$\frac{1}{2}$

Find the standard deviation of X.

a. 4 b. –1 c. 5 d. $\sqrt{\frac{74}{3}}$ e. 6

16. Find y such that the table below represents a possible probability distribution of a random variable X.

k	Pr(X = k)
0	.1
1	.2
2	.4
3	y

a. .6 b. .3 c. 1 d. .2 e. 0

17. An Olympic pistol shooter has $\frac{2}{3}$ chances of hitting the target at each shot. Find the probability that he will hit exactly 10 targets in a game of 15 shots.

a. $1 - \binom{15}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{5}$ b. $\binom{15}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{5}$ c. $\left(\frac{2}{3}\right)^{10}$

d. $\binom{15}{10}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{10}$ e. $1 - \left(\frac{1}{3}\right)^{5}$

18. The amount of milk contained in a gallon container is normally distributed with mean 128.2 ounces and standard deviation .2 ounces. What is the probability that a random bottle contains less than 128 ounces? a. .3085 b. .8413 c. .1587 d. .6915 e. 0.5

19. The length of the pregnancy in females of a certain species is normally distributed with mean 6 months and standard deviation $\frac{1}{2}$ months. What is the probability that the length of the pregnancy is between 6 and 7 months?

a. .3413 b. .4772 c. .0228 d. .8185 e. .8413

20. A dice is rolled 180 times. Use the normal approximation to estimate the probability of getting at least 27 fives.

a. .2743 b. .7257 c. 242 d. .758 e. .6915

21. Let C = $\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ and A = $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

If AB = C, find the entry in the second row and second column of B.

a. 1 b. 0 c. 6 d. 5 e. 2

22. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 3 & 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 2 & 1 \\ -1 & 4 & 0 & 3 \\ 3 & 1 & 1 & 5 \end{bmatrix}$. Find the entry in the 2nd row and 3rd column of AB. a. 2 b. 13 c. 1 d. 0 e. –1

23. A dice is rolled twice. What is the probability that the sum of the two numbers is less than or equal to 4? a. $\frac{1}{9}$ b. $\frac{5}{36}$ c. $\frac{1}{12}$ d. $\frac{1}{6}$ e. $\frac{7}{36}$

24. Let $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}$. Find the entry in the second row and second column of A^{-1}.

a. 1 b. –1 c. 3 d. 2 e. –2

25. Given that $A^{-1} = \begin{bmatrix} -1 & 3 \\ 1 & 2 \end{bmatrix}$, solve for y in the system of equations.

$A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$ a. 9 b. 11 c. 1 d. $\frac{14}{5}$ e. 3

26. Which of the following statements about the solution to the system.

$\begin{cases} 2x + 8y + 6z = 20 \\ 4x + 2y - 2z = -2 \\ -6x + 4y + 10z = 30 \end{cases}$ is true?

a. x = –2 b. x = –1 c. x = 1 d. The system does not have any solutions.
e. The system has infinitely many solutions.

27. A Markov chain has a transition matrix $\begin{bmatrix} .4 & .2 \\ .6 & .8 \end{bmatrix}$. If the initial probability distribution is $\begin{bmatrix} .5 \\ .5 \end{bmatrix}$, which of the following is the probability distribution two observations later?

a. $\begin{bmatrix} .26 \\ .74 \end{bmatrix}$ b. $\begin{bmatrix} 1.30 \\ .79 \end{bmatrix}$ c. $\begin{bmatrix} .34 \\ .66 \end{bmatrix}$ d. $\begin{bmatrix} .38 \\ .62 \end{bmatrix}$ e. $\begin{bmatrix} .24 \\ .76 \end{bmatrix}$

28. A chocolate factory produces a type of milk chocolate egg (MCE) and a type of dark chocolate egg (DCE). The production of each MCE requires 2 units of milk and 1 unit of cocoa butter. The production of each DCE requires 1 unit of milk and 1 unit of cocoa butter. The factory has a daily availability of 10 units of milk and 6 units of cocoa butter. The profit on each MCE is $7.5 and it is $5 for a DCE. How should the factory structure the daily production in order to maximize the profit?

a. No MCE and 5 DCE b. No MCE and 6 DCE c. 4 MCE and 2 DCE d. 2 MCE and 4 DCE
e. 4 MCE and 6 DCE

29. A bakery produces 200 loaves of bread each day, divided into 3 different kinds: white, rye and pumpernickel. At least half the daily production must be white bread and it is required that more rye than pumpernickel be produced. A daily consumption of at most 360 grams of yeast is allowed. A loaf of white bread requires 2 grams of yeast while a loaf of rye needs 2 grams and a loaf of pumpernickel needs 1 gram. Find the maximum daily profit possible for the bakery if profit on each loaf of bread is 30 cents for white, 20 cents for rye and 10 cents for pumpernickel.

a. $46 b. $48 c. $45 d. $50 e. $60

30. A city is experiencing a population movement towards the suburbs. Each year 7% of the city people move to the suburbs, whereas only 1% of those living in the suburbs move back to the city. In the long run, what proportion of people will end up living in the suburbs?

a. $\frac{7}{8}$ b. $\frac{6}{7}$ c. $\frac{3}{4}$ d. $\frac{4}{5}$ e. $\frac{2}{3}$