Formulas:

Compound Interest.

Compound amount $F = (1 + i)^n P$

present value $P = \frac{1}{(1 + i)^n} F$

Simple Interest

amount $A = (1 + n r) P$

annuity

$F = s_{n\rceil i} R$, $R = \frac{1}{s_{n\rceil i}} F$

$P = a_{n\rceil i} R$, $R = \frac{1}{a_{n\rceil i}} P$

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Problems (1), (2), and (3) refer to the following matrices:

$$A = \begin{pmatrix} .4 & .5 & 0 \\ 0 & .4 & 1 \\ .6 & .1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} .5 & 0 \\ .5 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & .8 & .2 & 0 \\ 0 & .1 & .8 & 1 \\ 0 & .1 & 0 & 0 \end{pmatrix}$$

$$D = \begin{pmatrix} 0 & 0 & .2 & 0 \\ .8 & .2 & 0 & 0 \\ .1 & .8 & .8 & 0 \\ .1 & 0 & 0 & 1 \end{pmatrix} \qquad E = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

1. a. A is absorbing b. E is absorbing c. C is absorbing
   d. B and D are absorbing e. none of them is absorbing

2. a. A is regular b. B is regular c. C is regular
   d. D is regular e. E is regular

3. The inverse matrix E^{-1} of E is

a. $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ b. $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ c. $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ d. $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$ e. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

4. $E^2 - B =$

a. $\begin{pmatrix} -.5 & 0 \\ 0 & 0 \end{pmatrix}$ b. $\begin{pmatrix} .5 & 0 \\ -.5 & 0 \end{pmatrix}$ c. $\begin{pmatrix} .5 & 1 \\ .5 & 0 \end{pmatrix}$

d. $\begin{pmatrix} .5 & -1 \\ -.5 & 0 \end{pmatrix}$ e. $\begin{pmatrix} .5 & 0 \\ 0 & -.5 \end{pmatrix}$

5. The stable matrix of $A = \begin{pmatrix} 1 & 0 & 0 & .2 \\ 0 & 1 & 0 & .2 \\ 0 & 0 & 1 & .4 \\ 0 & 0 & 0 & .2 \end{pmatrix}$ is

a. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ b. $\begin{pmatrix} .2 & .2 & .2 & .2 \\ .2 & .2 & .2 & .2 \\ .4 & .4 & .4 & .4 \\ .2 & .2. & 2. & 2 \end{pmatrix}$ c. $\begin{pmatrix} 1 & 0 & 0 & \frac{1}{.2} \\ 0 & 1 & 0 & \frac{1}{.2} \\ 0 & 0 & 1 & \frac{1}{-4} \\ 0 & 0 & 0 & 0 \end{pmatrix}$

d. $\begin{pmatrix} 1 & 0 & 0 & .5 \\ 0 & 1 & 0 & .5 \\ 0 & 0 & 1 & .25 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ e. $\begin{pmatrix} 1 & 0 & 0 & \frac{1}{4} \\ 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix}$

6. Let $A = \begin{pmatrix} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{pmatrix}$ then the stable matrix is

a. $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ b. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ c. $\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

d. $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ e. $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

7. The stable distribution of the regular stochastic matrix $A = \begin{pmatrix} .4 & .3 \\ .6 & .7 \end{pmatrix}$ is

a. $\begin{pmatrix} .4 \\ .6 \end{pmatrix}$ b. $\begin{pmatrix} .3 \\ .7 \end{pmatrix}$ c. $\begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}$ d. $\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$ e. $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

8. A car rental agency has outlets in 3 cities. If a car is rented in city 1, there is a 70% chance that it will be returned to city 1 and a 20% chance that it will be returned to city 2. If a car is rented in city 2, there is a 50% chance that it will be returned to city 2 and a 30% chance that it will be returned to city 3. If a car is rented in city 3, there is a 40% chance that it will be returned to city 3 and a 20% chance that it will be returned to city 1. What is the stable distribution of this car rental agency?

a. $\begin{pmatrix} \frac{15}{45} \\ \frac{15}{45} \\ \frac{15}{45} \end{pmatrix}$ b. $\begin{pmatrix} \frac{18}{45} \\ \frac{16}{45} \\ \frac{11}{45} \end{pmatrix}$ c. $\begin{pmatrix} \frac{20}{45} \\ \frac{8}{45} \\ \frac{17}{45} \end{pmatrix}$

d. $\begin{pmatrix} \frac{40}{45} \\ \frac{2}{45} \\ \frac{3}{45} \end{pmatrix}$ e. $\begin{pmatrix} \frac{1}{45} \\ \frac{23}{45} \\ \frac{21}{45} \end{pmatrix}$

9. A savings account pays 8% annual interest compounded quarterly. How much should you deposit in the account now so that you will have $10,000 in the account after 4 years.

a. $5,3650.13 b. $7,365.01 c. $1,413.26

d. $8,528.21 e. $7,284.46

10. How much should you deposit now into an account paying 12% annual interest compounded monthly so that you can withdraw $100 at the end of each month from the account for 4 years?

a. $3,901.26 b. $4,520.25 c. $4,8000.01

d. $3,797.40 e. $2,633.34

11. $1 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9 = ?$

a. 2^{10} b. $2^{10} - 1$ c. $2^{10} + 1$ d. 2^{11} e. $2^{11} - 1$

12. Consider the following saving accounts passbook:

Date	Deposit	Interest	Balance
Jan 1, 1994	$5,000		$5,000
July 1, 1994		$50.00	$5,050
Jan 1, 1995		$50.50	$5,100.50

What is the annual interest rate of this account?

a. $\frac{1}{2}$ % b. 1% c. 2% d. 3% e. 4%

13. Joan deposits $10,000 in an account paying 6% annual interest compounded monthly. How much can she withdraw from the account each month for 5 years?

a. $207.58 b. $193.33 c. $156.66

d. $171.68 e. $222.44

14. Tom made a deposit into an account paying 8% annual interest compounded quarterly. At the end of 9 years, the account is worth $100,000. How much of the $100,000 would be interest?

a. $39,232.85 b. $19,232.85 c. $33,333.33

d. $60,135.25 e. $50,977.68

15. Bill deposits $1000 each quarter into an account paying 6% annual rate compounded quarterly. How much interest will Bill have earned at the end of 5 years?

a. $5,123.11 b. $17,168.64 c. $23,123.67

d. $3,123.67 e. $1,716.86

16. How long would it take for an investment of $500 to double at an annual rate of 5% simple interest?

a. 1 year b. 5 years c. 10 years

d. 20 years e. 30 years

17. How much should you deposit each month into an account paying 9% annual interest compounded monthly so that you will have $150,000 at the end of ten years?

a. $918.68 b. $775.14 c. $859.40

d. $10,351.40 e. $655.34