1. In a 10-team soccer conference, each team plays every other team exactly once. How many games must be played?

a. 45 b. 90 c. 99 d. 100 e. 10!

2. A class of 100 students is split into 3 groups: group 1 is to have 20 students, group 2 is to have 30 students and group 3 is to have 50 students. How many ways are there to do this?

a. $\binom{100}{20}\binom{80}{30}$ b. $\binom{100}{20}\binom{100}{30}\binom{100}{50}$ c. $\binom{100}{20} + \binom{100}{30} + \binom{100}{50}$

d. $20 \cdot 30 \cdot 50$ e. $20 + 30 + 50$

3. At Smith College, 260 mathematics majors are surveyed about 3 courses: finite math (F), Calculus (C), and Algebra (A). It is found that 52 students take all 3 courses and that 100 take A, 200 take C, 165 take F, 57 take A and C, 125 take C and F, 82 take A and F. How many students take none of the 3 courses?

a. 5 b. 7 c. 10 d. 13 e. 25

4. A fast food place offers a pancake combo, consisting of a basic pancake with a choice of up to 3 extras from a list of 8. How many different pancake combos are possible?

a. 56 b. 93 c. 100 d. 336 e. 256

5. How many 5-digit numbers can be made with the digits 1 through 8 if no digit is repeated?

a. 8^5 b. $\frac{8!}{5!\,3!}$ c. $\frac{8!}{5!}$ d. $\frac{8!}{3!}$ e. 2^5

6. An urn contains 8 green balls and 6 red balls. Five balls are selected at random. Find the probability that exactly 3 of the balls are red.

a. $\frac{\binom{6}{3}}{\binom{14}{5}}$ b. $1 - \frac{\binom{6}{3}}{\binom{14}{5}}$ c. $\frac{\binom{6}{3}\binom{8}{2}}{\binom{14}{5}}$

d. $1 - \frac{\binom{8}{2}}{\binom{14}{5}}$ e. $\frac{\binom{6}{3}}{\binom{14}{5}} + \frac{\binom{8}{2}}{\binom{14}{5}}$

7. Suppose that, in a certain experiment, the events E and F are independent. If $Pr(E) = Pr(F) = \frac{1}{2}$, what is $Pr(E \cup F)$?

a. $\frac{2}{3}$ b. $\frac{3}{4}$ c. 1 d. $\frac{7}{8}$ e. not enough information

8. A study finds that 20% of all inhabitants of the western part of Scotland suffer from heart disease, but only 10% of the inhabitants of the eastern part of Scotland do. 30% of the people of Scotland live in the western part, and 70% live in the eastern part. Suppose a Scottish citizen is chosen at random and is found to have heart disease. What is the probability that he/she comes from the western part of Scotland?

a. $\frac{2}{3}$ b. $\frac{3}{10}$ c. $\frac{6}{13}$ d. $\frac{6}{70}$ e. $\frac{6}{7}$

9. Two cards are drawn from a standard deck of 52 cards. What is the probability that the first card is an ace and the second is a King if the first card is replaced before the second is drawn?

a. $\frac{4}{52} + \frac{3}{52}$ b. $\frac{4}{52} + \frac{3}{51}$ c. $\left(\frac{4}{52}\right)^2$

d. $\left(\frac{4}{52}\right)\left(\frac{5}{51}\right)$ e. $\left(\frac{4}{52}\right)\left(\frac{3}{51}\right)$

10. Suppose that E and F are events in an experiment, and $\Pr(E) = \frac{1}{4}$, $\Pr(F) = \frac{1}{2}$, $\Pr(E \cup F) = \frac{3}{4}$. What is $\Pr(E|F)$?

a. 1 b. $\frac{1}{2}$ c. $\frac{1}{4}$ d. 0 e. $\frac{1}{3}$

11. An urn contains 5 red balls and 5 white balls. 3 balls are drawn from the urn at random, one at a time and without replacement. What is the probability that the first ball drawn is red and the second and third are white?

a. $\frac{1}{9}$ b. $\frac{1}{8}$ c. $\frac{5}{12}$ d. $\frac{2}{25}$ e. $\frac{5}{36}$

12. A pair of fair dice is rolled 3 times. Find the probability that the dice add up to 7 each time.

a. $\frac{1}{12}$ b. $\frac{1}{6}$ c. $\left(\frac{1}{12}\right)$ d. $\left(\frac{1}{6}\right)^3$ e. $\frac{1}{2}$

13. In a certain factory, an old machine produces bolts of which 10% are defective. What is the probability that, in a random sample of 80 bolts produced by the machine, at least 3 are defective?

a. $1 - (.9)^{80} - \binom{80}{1} (.1) (.9)^{79} - \binom{80}{2} (.1)^2 (.9)^{78}$

b. $1 - (.9)^{80} - (.1) (.9)^{79} - (.1)^2 (.9)^{78}$

c. $\binom{80}{3} (.1)^3 (.9)^{77}$

d. $(.1)^3 (.9)^{77}$

e. $(.9)^{80} + \binom{80}{1} (.1) (.9)^{79} + \binom{80}{2} (.1)^2 (.9)^{78} + \binom{80}{3} (.1)^3 (.9)^{77}$

14. The weight of a certain type of car (when it leaves the factory) is normally distributed with mean 998 kg and standard deviation .8 kg. Find the probability that a new car of this type chosen at random weighs between 997 kg and 999 kg.

a. .8944 b. .7698 c. .7888 d. .9876 e. .9938

15. Five fair coins are tossed simultaneously and the number X of heads is observed. What is the variance of the random variable X?

a. $\sqrt{\frac{5}{4}}$ b. $\frac{5}{2}$ c. $\frac{5}{4}$ d. $\frac{55}{25} - \frac{4}{25}$ e. 1

16. The probability that a certain surgical operation is successful is 0.8 (it is a binomial distribution). If the operation is performed on 100 people, find the probability that 70 or more operations are successful (use normal distribution to approximate the binomial distribution):

a. .9938 b. .0062 c. .5 d. .8944 e. .1056

17. A bag contains three \$1 bills, two \$5 bills, and one \$10 bill. One bill is selected at random. If X denotes the denomination of the selected bill, find the expected value E(X).

a. $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$ b. $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{6}$ c. $\left(\frac{1}{2}\right)^3 \left(\frac{1}{3}\right)^2 \left(\frac{1}{6}\right)$

d. $\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{6}\right)$ e. $1 \cdot \frac{1}{2} + 5 \cdot \frac{1}{3} + 10 \cdot \frac{1}{6}$

18. A random variable X has the following probability distribution:

k	Pr(X = k)
−10	1/3
0	1/3
1	1/6
2	1/6

What is the expected value E(X)?

a. $-\frac{13}{4}$ b. $-\frac{7}{4}$ c. $-\frac{17}{6}$ d. $\frac{1}{4}$ e. $-\frac{7}{3}$

19. The stable matrix of the absorbing stochastic matrix $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{4} & \frac{1}{2} \end{bmatrix}$ is

a. $\begin{bmatrix} 1 & 0 & 1 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 & \frac{1}{2} & 1 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ e. $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

20. The transition matrix of a Markov Process is given by the matrix $\begin{bmatrix} .7 & .1 \\ .3 & .9 \end{bmatrix}$. The stable distribution of this process is:

a. $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$ b. $\begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$ c. $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$ d. $\begin{bmatrix} \frac{2}{5} \\ \frac{3}{5} \end{bmatrix}$ e. $\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$

21. The transition matrix of a Markov process is given by the matrix.

$$A = \begin{bmatrix} .1 & 0 & 0 \\ .2 & .5 & 0 \\ .7 & .5 & 1 \end{bmatrix}$$

The matrix A is

a. regular b. absorbing c. regular and absorbing

d. regular but not absorbing e. neither regular nor absorbing

22. Calculate the amount at the end of 5 years if $2,000 is invested at 5% simple interest.

a. 2,500 b. 2,250 c. 2,025 d. 2,750 e. 3,000

23. Let $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ be the transition matrix of a Markov Process. If the distribution of the current generation is $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$. Then the distribution of the next generation is

a. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ b. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ c. $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$ d. $\begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$ e. $\begin{bmatrix} \frac{1}{8} \\ \frac{1}{8} \end{bmatrix}$

24. Which of the payoff matrices

$A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 1 \\ 2 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 2 \\ 6 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$

have saddle points?

a. only B
b. A, C and D but not B
c. only A
d. only A and D
e. all

25. Suppose that a game has payoff matrix $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$. What is the optimal strategy for Player R?

a. [1/3 2/3]
b. [1/2 1/2]
c. [1 0]
d. [0 1]
e. [1/4 3/4]

26. Rick (R) and Catherine (C) are playing "two-finger morra", a game in which each player first makes a fist then, at the count of three, extends either 1 or 2 fingers (F). If the sum of the number of fingers is even, Rick gets that amount of money (either $2 or $4), and if the number is odd, Catherine gets $3. The pay-off matrix of this game is given by

a.

		C: 1F	C: 2F
R	1 F	2	3
	2 F	3	4

b.

		C: 1F	C: 2F
R	1 F	–4	3
	2 F	3	–2

c.

		C: 1F	C: 2F
R	1 F	4	–3
	2 F	–3	2

d.

		C: 1F	C: 2F
R	1 F	–3	4
	2 F	2	–3

e.

		C: 1F	C: 2F
R	1 F	2	–3
	2 F	–3	4

27. Ted needs $10,000 four years from now. How much should he invest now (one lump sum) in a savings account paying 6% annual interest compounded monthly?

a. $7,870.99 b. $1,633.39 c. $2,488.51

d. $2,633.39 e. $8,356.45

28. Mr. Rich takes out a 30-year $300,000 mortgage at 9% annual interest, compounded monthly, with payments made monthly. What is the unpaid balance at the end of twenty years?

a. $2413.87 b. $190,555.72 c. $109,444.28

d. $100,000 e. $290,000.15

29. Sue needs $10,000 four years from now in order to pay off a loan. How much must she save each quarter for the next four years if interest rates are 8% compounded quarterly?

a. $490.22 b. $386.53 c. $326.02

d. $536.50 e. $192.32

30. Mr. Smart purchased a car for $1000 down payment plus monthly payments of $300 for 3 years, at the annual interest rate of 18% compounded monthly. What is the purchase price (present value) of the car?

a. $108,000 b. $118,000 c. $97,000 d. $8298.21 e. $9298.21