

Formulas:

$$F = (1 + i)^n P \quad F = s_{ni} R \quad P = a_{ni} R$$

$$s_{ni} = \frac{(1 + i)^n - 1}{i} \quad a_{ni} = \frac{(1 + i)^n - 1}{i(1 + i)^n}$$

If $A = \begin{bmatrix} I & S \\ 0 & R \end{bmatrix}$ is an absorbing stochastic matrix then the stable matrix of A is $\begin{bmatrix} I & S(I - R)^{-1} \\ 0 & 0 \end{bmatrix}$. [Note: The identity matrix I in $(I - R)^{-1}$ is chosen to be the same size as R in order to make the matrix subtraction permissible.]

1. Consider the matrix $A = \begin{bmatrix} 0.2 & 0.2 & 0.5 \\ 0.3 & 0.3 & 0.3 \\ 0.1 & 0.5 & 0.2 \end{bmatrix}$. Which of the following statements about A is true?

- a. A is stochastic.
- b. A is stochastic and regular.
- c. A is stochastic and absorbing.
- d. A is stochastic, regular but not absorbing.
- e. A is not stochastic.

2. Consider the stochastic matrix $A = \begin{bmatrix} 1 & 0.2 \\ 0 & 0.8 \end{bmatrix}$. Then A is:

- a. absorbing and regular
- b. absorbing but not regular
- c. regular but not absorbing
- d. stable but not absorbing
- e. neither absorbing nor regular

3. Let $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$ be the transition matrix of a Markov process. If the distribution

matrix of the current generation is $\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$. Then the distribution of the next generation

is:

- a. $\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$
- b. $\begin{bmatrix} \frac{2}{9} \\ \frac{4}{9} \\ \frac{4}{9} \end{bmatrix}$
- c. $\begin{bmatrix} \frac{3}{9} \\ \frac{2}{9} \\ \frac{4}{9} \end{bmatrix}$
- d. $\begin{bmatrix} \frac{2}{9} \\ \frac{3}{9} \\ \frac{4}{9} \end{bmatrix}$
- e. $\begin{bmatrix} \frac{1}{9} \\ \frac{2}{9} \\ \frac{4}{9} \end{bmatrix}$

4. A Markov process with three states has transition matrix

| | | Current State | | |
|------------|-----|---------------|-----|-----|
| | | I | II | III |
| Next State | I | 0.2 | 0.3 | 0.4 |
| | II | 0.1 | 0.2 | 0 |
| | III | 0.7 | 0.5 | 0.6 |

The probability of proceeding from (current) state III to (next) state I is:

- a. 0.2 b. 0.3 c. 0.4 d. 0.5 e. 0.7

5. The stable distribution $\begin{bmatrix} x \\ y \end{bmatrix}$ of the matrix $A = \begin{bmatrix} 0.5 & 0.6 \\ 0.5 & 0.4 \end{bmatrix}$ is:

- a. $\begin{bmatrix} 6 \\ 11 \\ 5 \\ 11 \end{bmatrix}$ b. $\begin{bmatrix} 7 \\ 11 \\ 4 \\ 11 \end{bmatrix}$ c. $\begin{bmatrix} 4 \\ 9 \\ 5 \\ 9 \end{bmatrix}$ d. $\begin{bmatrix} 0.6 \\ 0.4 \end{bmatrix}$ e. $\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$

6. The stable matrix of the transition matrix $A = \begin{bmatrix} 1 & 0 & 0.4 \\ 0 & 1 & 0.5 \\ 0 & 0 & 0.1 \end{bmatrix}$ is

- a. $\begin{bmatrix} 1 & 0 & 0.36 \\ 0 & 1 & 0.45 \\ 0 & 0 & 0 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & \frac{4}{9} \\ 0 & 1 & \frac{5}{9} \\ 0 & 0 & 0 \end{bmatrix}$ c.

$\begin{bmatrix} 0 & 0 & \frac{4}{9} \\ 0 & 0 & \frac{5}{9} \\ 0 & 0 & 0 \end{bmatrix}$

- e. $\begin{bmatrix} 1 & 0 & \frac{5}{9} \\ 0 & 1 & \frac{4}{9} \\ 0 & 0 & 0 \end{bmatrix}$ e. $\begin{bmatrix} 1 & 0 & \frac{1}{10} \\ 0 & 1 & \frac{9}{10} \\ 0 & 0 & 0 \end{bmatrix}$

7. A professor's exams are either easy or hard. If the exam was easy last time, it will be easy this time with a 40% probability. If it was hard last time, it will be easy this time with a 70% probability. The matrix of this Markov process is given by:

- a. $\begin{matrix} & \begin{matrix} \text{current} \\ \text{easy} & \text{hard} \end{matrix} \\ \begin{matrix} \text{easy} \\ \text{hard} \end{matrix} & \begin{bmatrix} 0.7 & 0.4 \\ 0.3 & 0.6 \end{bmatrix} \end{matrix}$ b. $\begin{matrix} & \begin{matrix} \text{current} \\ \text{easy} & \text{hard} \end{matrix} \\ \begin{matrix} \text{easy} \\ \text{hard} \end{matrix} & \begin{bmatrix} 0.3 & 0.4 \\ 0.7 & 0.6 \end{bmatrix} \end{matrix}$ c. $\begin{matrix} & \begin{matrix} \text{current} \\ \text{easy} & \text{hard} \end{matrix} \\ \begin{matrix} \text{easy} \\ \text{hard} \end{matrix} & \begin{bmatrix} 0.6 & 0.3 \\ 0.4 & 0.7 \end{bmatrix} \end{matrix}$

- d. $\begin{matrix} & \begin{matrix} \text{current} \\ \text{easy} & \text{hard} \end{matrix} \\ \begin{matrix} \text{easy} \\ \text{hard} \end{matrix} & \begin{bmatrix} 0.4 & 0.3 \\ 0.6 & 0.7 \end{bmatrix} \end{matrix}$ e. $\begin{matrix} & \begin{matrix} \text{current} \\ \text{easy} & \text{hard} \end{matrix} \\ \begin{matrix} \text{easy} \\ \text{hard} \end{matrix} & \begin{bmatrix} 0.4 & 0.7 \\ 0.6 & 0.3 \end{bmatrix} \end{matrix}$

8. Which of the matrices are regular:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

- a. A_1 b. A_2 c. A_3 d. A_4 e. A_3 and A_4

9. How much money must be deposited now in order to have \$10,000 after 10 years if interest is paid at a 6% annual rate compounded monthly?

- a. \$928.92 b. \$5,496.33 c. \$3,029.95
d. \$7,413.72 e. \$5,904.50

10. One thousand dollars is deposited in an account at 8% annual rate compounded quarterly for 2 years. The amount of interest earned during that time is

- a. \$60.30 b. \$90.49 c. \$171.66
d. \$214.98 e. \$306.22

11. How much should Mary save each month to have \$20,000 for the down payment to buy a house in 5 years if annual interest rate is 6% compounded monthly?

- a. \$692.15 b. \$511.64 c. \$399.68
d. \$286.66 e. \$201.57

12. If the annual interest rate is 12% compounded daily, the rate per period i is:

- a. 1% b. $\frac{12}{365}$ % c. $\frac{1}{2}$ % d. 3% e. 12%

13. How much should Jim deposit in an account paying 12% annual rate compounded monthly so that his son can withdraw \$100 at the end of each month for 9 months?

- a. \$900.00 b. \$890.22 c. \$856.60
d. \$1072.36 e. \$998.53

14. If you deposit \$10,000 into an account paying 8% annual interest compounded quarterly. How much can you withdraw at the end of each quarter year for 5 years so that balance is zero at the end of 5 years?

- a. \$611.57 b. \$6,115.67 c. \$523.72
d. \$5,237.23 e. \$557.66

15. What is the monthly payment on a 30 year \$100,000 mortgage at 12% annual rate compounded monthly?

- a. \$1543.31 b. \$1507.08 c. \$1101.08
d. \$1,053.22 e. \$1,028.61

16. Dan took a loan to buy a car. If the interest rate is 18% compounded monthly and the monthly payment is \$200 for 4 years. What is the amount of the loan?

- a. \$6,808.51 b. \$7,808.51 c. \$5,532.14
d. \$6,532.14 e. \$9,172.44

17. Janet took out a loan in the amount of \$600. If the annual interest rate is 18% compounded monthly. How much interest did Janet pay at the end of the first month?

- a. \$108 b. \$10.8 c. \$1.08 d. \$90 e. \$9

18. Sam deposits \$1,000 per month for 3 years at 12% annual interest rate compounded monthly. How much will Sam have at the end of 3 years?

- a. \$39,336.10 b. \$43,076.88 c. \$36,000
d. \$36,360 e. \$47,275.96

19. If $A = \begin{bmatrix} 0.4 & 1 \\ 0.6 & 0 \end{bmatrix}$ then $A^2 =$

- a. $\begin{bmatrix} 0.22 & 0.4 \\ 0 & 0.6 \end{bmatrix}$ b. $\begin{bmatrix} 0.16 & 1 \\ 0.36 & 0 \end{bmatrix}$ c.
 $\begin{bmatrix} 1.4 & 1 \\ 0.6 & 0 \end{bmatrix}$
d. $\begin{bmatrix} 0.76 & 0.4 \\ 0.24 & 0.6 \end{bmatrix}$ e. $\begin{bmatrix} 0.8 & 2 \\ 1.2 & 0 \end{bmatrix}$

20. $1 - 3 + 3^2 - 3^3 + 3^4 - \dots + 3^{98}$

- a. $3^{99} - 1$ b. $\frac{3^{99} - 1}{3}$ c. $\frac{(-3)^{99} - 1}{-4}$
d. $(-3)^{99}$ e. 3^{99}

