Formulas:

$$
\begin{array}{rl}
F=(1+i)^{n} P \quad F=s_{n\rceil i} R & P=a_{n\rceil i} R \\
s_{n\rceil i}=\frac{(1+i)^{n}-1}{i} & a_{n\rceil i}=\frac{(1+i)^{n}-1}{i(1+i)^{n}} \\
\text { If } \quad A=\left[\begin{array}{ll}
a & b \\
c & d
\end{array}\right] \text { then } \quad A^{-1}=\left[\begin{array}{cc}
\frac{d}{a d-b c} & \frac{-b}{a d-b c} \\
\frac{-c}{a d-b c} & \frac{a}{a d-b c}
\end{array}\right]
\end{array}
$$

1. Let $A=\{c, d, e\}$, let $B=\{x, y\}$, and let the universal set $U=\{c, d, e, x, y, z\}$. Then $\left(A^{\prime} \cap B^{\prime}\right)^{\prime}=$
a) $\{z\}$
b) $\{d, e\}$
c) $\{c, d, e, x, y\}$
d) $U$
e) $\varnothing$
2. Out of 100 people in a certain class, 70 people did well on the first test, 60 did well on the second test, and 50 did well on both tests. How many people did well on neither test?
a) 60
b) 30
c) 50
d) 20
e) 40
3. A company has 10 cars and 10 garages. Each night the cars are all put away-exactly one to a garage. In how many ways can the cars be put away?
a) 10 !
b) $2^{10}$
c) $\binom{10}{1}$
d) $P(10,5)$
e) $10^{2}$
4. A person tosses a fair coin 3 times. What is the probability of getting exactly one tail out of the 3 tosses?
a) $\frac{1}{8}$
b) $\frac{3}{8}$
c) $\frac{5}{8}$
d) $\frac{1}{3}$
e) $\frac{2}{8} \cdot \frac{1}{8}$

In the next 2 problems there is a house with exactly 400 books, 320 are math books and 80 are novels (which are not math books.)
5. How many different samples of 10 books can be chosen?
a) $\binom{400}{10}$
b) $P(400,10)$
c) $\frac{320 \cdot 80}{6!}$
d) $400^{10}$
e) $\binom{320}{8}\binom{80}{2}$
6. How many samples of 10 books contain no math books?
a) $10^{320}$
b) $400^{10}-80^{10}$
c) $80^{10}$
d) $\binom{400}{10}-\binom{320}{10}$
e) $\binom{80}{10}$

In the next 3 problems, let $S$ be a sample space with $E, F, G$ events associated to S. Assume $P(E)=0.5, P(F)=0.6, P(E \cap F)=0.2$ and $P(G)=0.4$.
7. If $E$ and $G$ are independent the $P(E \cap G)=$
a) not enough information
b) $\frac{1}{5}$
c) $\frac{4}{5}$
d)
$\begin{array}{ll}\frac{1}{2} & \text { e) } \frac{2}{5}\end{array}$
8. $\quad P(E I F)=$
a) 0.1
b) 0.12
c) 0.4
d) $\frac{2}{4}$
e) $\frac{1}{3}$
9. $\quad \mathrm{P}\left(\mathrm{F}^{\prime}\right)=$
a) not enough information
b) 0.6
c) 0.24
d) 0.4
e) 0.2

Problems 10 and 11 refer to the following 2 matrices

$$
\text { Let } M=\left[\begin{array}{ll}
1 & 3 \\
2 & 7
\end{array}\right], \quad N=\left[\begin{array}{ccc}
2 & 0 & -1 \\
1 & 1 & 0
\end{array}\right]
$$

10. Find $\mathrm{M} \cdot \mathrm{N}$
a) $\left[\begin{array}{rrr}2 & 0 & -1 \\ 5 & 7 & 0\end{array}\right]$
b) $\left[\begin{array}{rrr}5 & 3 & -1 \\ 11 & 7 & -2\end{array}\right]$
c) $\left[\begin{array}{ll}2 & 5 \\ 0 & 7 \\ 1 & 0\end{array}\right]$
d) $\left[\begin{array}{rrr}-1 & 3 & 5 \\ 0 & 7 & 11\end{array}\right]$
e) They can't be
multiplied
11. Find $\mathrm{M}^{-1}$
a) $\left[\begin{array}{rr}7 & -3 \\ -2 & 1\end{array}\right]$
b) $\left[\begin{array}{ll}-3 & -14\end{array}\right]$
c) $\left[\begin{array}{ll}2 & 7 \\ 1 & 3\end{array}\right]$
d) $\left[\begin{array}{rr}3 & -7 \\ -1 & 2\end{array}\right]$
e) $\left[\begin{array}{l}-14 \\ -3\end{array}\right]$
12. Suppose $x$ and $y$ satisfy the two equations $x+4 y=2 \quad 2 x+6 y=1 \quad$ What is $y$ ?
a) 2
b) -1
c) $\frac{3}{2}$
d) $-\frac{5}{2}$
e) not enough information.
13. Use the normal approximation to the binomial distribution to estimate the probability of getting 55 or more heads in 100 tosses of a fair coin.
a) .6179
b) .3821
c) .1841
d) .0359
e) .4500
14. The random variable $Y$ has the probability distribution shown.

| $Y=k$ | -3 | 0 | 3 | 5 |
| :--- | :---: | :---: | :---: | :---: |
| $\operatorname{Pr}(Y=k)$ | .1 | .6 | .1 | .2 |

What is the expected value of $Y$ ?
a) 0 b) -.5
c) .5
d) 1
e)

## 1.6

15. An experiment consists of rolling a fair die 10 times and counting the number of ones. What is the probability of getting 1 or 2 ones?
a) $\binom{10}{1}+\binom{10}{2}$
b) $\binom{10}{1}\left(\frac{1}{6}\right)^{1}+\binom{10}{2}\left(\frac{1}{6}\right)^{2}$
c) $\binom{10}{1}\left(\frac{1}{6}\right)^{10}+\binom{10}{2}\binom{1}{6}^{10}$


16. Let $X$ denote a random variable having a normal distribution with $\mu=10$
and $\sigma=4$. Find $\operatorname{Pr}(10 \leq X \leq 14)$.
a) .3413
b) .5000
c) .0060
d) .6915
e) .3085
17. Suppose the length of the Madagascar hissing cockroach is normally distributed with $\mu=40$ millimeters and $\sigma=5$. What is the probability of a Madagascar hissing cockroach having a
length of 45 or more millimeters?
a) .0228
b) .1587
c) .3085
d) .3821
e) . 6179
18. A survey is conducted and it is found that $60 \%$ of Indiana residents can name the capital of Florida. What is the probability that exactly 4 out of 7 randomly chosen Indiana residents can name the capital of Florida?
a) $\binom{4}{7}(.6)$
b) $\binom{7}{4}(.6)(.4)$
c) $\binom{7}{4}(.6)^{7}$
d) $\binom{7}{4}$
(.6) ${ }^{4}-\binom{7}{3}$
$(.4)^{3}$
e)
$\binom{7}{4}(.6)^{4}(.4)^{3}$
19. Dan invested $\$ 10,000$ at $12 \%$ annual interest compounded monthly. How much will he have at the end of 15 years?
a) $16,678.34$
b) $59,958.02$ c) 49,958.02
d) $83,321.66$
e) $34,548.15$
20. Sue took out a 25 -year $\$ 60,000$ mortgage at $6 \%$ annual interest compounded monthly.
What is the monthly payment?
a) $\$ 144.30$
b) $\$ 155.21$
c) $\$ 386.58$
d)
\$692.99
e) $\$ 865.81$
21. At the end of every 3 months Jason deposits $\$ 100$ into a savings account receiving $6 \%$ annual interest compounded quarterly. How much will Jason have in the account at the end of 5
years?
a) $5,331.28$
b) $6,977.00$
c) $4,324.46$
d) 1,716.86
e) $2,312.37$
22. How much money must you deposit now into a savings account receiving $8 \%$ annual interest rate compounded quarterly in order to be able to withdraw $\$ 2000$ at the end of each quarter
year for 12 years?
a) $61,346.24$
b) $68,085.11$
c) $158,707.01$
d) $50,977.68 \mathrm{e})$ 69,521.77
23. George took out a loan in the amount of $\$ 563$. He paid off the loan in 5 months with monthly payments of $\$ 116$. How much interest did he pay?
a) $\$ 56.30$
b) $\$ 116$
c) $\$ 11.60$
d) $\$ 44.7$
e) $\$ 17$
24. Helen would like to buy a $\$ 50,000$ recreational vehicle when she retires in 10 years. How much should she deposit at the end of each month into an account receiving $12 \%$ annual interest compounded monthly so that she will have enough money to purchase the vehicle?
a) $\$ 348.50$
b) $\$ 717.35$
c) $\$ 555.10$
d) $\$ 217.35$
e) $\$ 331.29$
25. Find the stable distribution $\left[\begin{array}{l}x \\ y\end{array}\right]$ of the matrix $\left[\begin{array}{ll}.2 & .4 \\ .8 & .6\end{array}\right]$.
a) $\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{l}\frac{1}{4} \\ \frac{3}{4}\end{array}\right]$
b) $\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{l}\frac{1}{2} \\ \frac{1}{2}\end{array}\right]$
c) $\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{l}\frac{1}{3} \\ \frac{2}{3}\end{array}\right]$
d) $\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{l}\frac{2}{3} \\ \frac{1}{3}\end{array}\right]$
e) $\left[\begin{array}{l}x \\ y\end{array}\right]=\left[\begin{array}{l}\frac{1}{4} \\ \frac{1}{4}\end{array}\right]$
26. If $\left[\begin{array}{ll}\frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4}\end{array}\right]$ is the transition matrix of a Markov process and the initial distribution is $\left[\begin{array}{l}0 \\ 1\end{array}\right]$. Then the next two distributions are:
a) $\left[\begin{array}{l}\frac{1}{4} \\ \frac{3}{4}\end{array}\right]$ and $\left[\begin{array}{l}\frac{2}{3} \\ \frac{1}{3}\end{array}\right]$
b) $\left[\begin{array}{l}\frac{1}{2} \\ \frac{1}{2}\end{array}\right]$ and $\left[\begin{array}{l}\frac{2}{3} \\ \frac{1}{3}\end{array}\right]$
c) $\left[\begin{array}{l}\frac{1}{4} \\ \frac{3}{4}\end{array}\right]$ and $\left[\begin{array}{l}\frac{1}{2} \\ \frac{1}{2}\end{array}\right]$
d) $\left[\begin{array}{l}\frac{1}{4} \\ \frac{3}{4}\end{array}\right]$ and $\left[\begin{array}{l}\frac{5}{16} \\ \frac{11}{16}\end{array}\right]$
e) $\left[\begin{array}{l}0 \\ 1\end{array}\right]$ and $\left[\begin{array}{l}\frac{1}{2} \\ \frac{1}{2}\end{array}\right]$
27. Let $\left[\begin{array}{lll}0 & \frac{1}{3} & \frac{1}{3} \\ 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3}\end{array}\right]$ be the transition matrix of a Markov process. Then the matrix is:
a) regular but not absorbing
b) regular and absorbing
c) absorbing but not regular
d) neither absorbing nor regular
e) none of the above
28. Which of the following statements are impossible:
(1) A graph with 7 vertices each of degree 5 .
(2) A connected graph with 5 vertices and 4 edges.
(3) A connected graph with 5 vertices and 3 edges.
(4) A graph with 4 vertices and 4 edges.
a) (1), (3) and (4)
b) (1) and (3)
c) (2) and (3)
d) (2) and (4)
e) (1) and (2)
29. A graph has 4 vertices, of degrees $2,3,4,5$ respectively. What is the number of edges?
a) 10
b) 6
c) 4
d) 7
e) not enough information
30. Which of the following graphs contain an Euler circuit?

3) 


a) 1 and 2
b) 1,2 and 3
c) 1 and 3
d) 2 and 3
e) 1

