

1. Let $S = \{a, b, c, d, e\}$ be a sample space. How many events are associated with S ?

- (a) 10 (b) 16 (c) 32 (d) 25 (e)

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2. Let E and F be events for which $\Pr(E) = .3$, $\Pr(F) = .6$, $\Pr(E \cap F) = .1$. Find $\Pr(E \cup F)$.

- (a) .2 (b) .3 (c) .7 (d) .8 (e) .9

3. An urn contains 6 white balls and 4 green balls. A sample of 2 balls is selected at random. What is the probability of selecting only white balls?

- (a) $\frac{\binom{6}{2}}{\binom{4}{2}}$ (b) $\frac{\binom{6}{2} \binom{4}{2}}{\binom{10}{2}}$ (c) $\frac{\binom{6}{2} \binom{5}{1}}{\binom{10}{2}}$
(d) $\frac{\binom{4}{0} + \binom{6}{2}}{10}$ (e) $\frac{\binom{6}{2}}{\binom{10}{2}}$

4. Three people are chosen at random. What is the probability that at least 2 of them were born on the same day of the week?

- (a) $\frac{3}{7}$ (b) $\frac{16}{49}$ (c) $\frac{2}{7}$ (d) $\frac{21}{49}$ (e) $\frac{6}{49}$

5. A coin is tossed 5 times. What is the probability of obtaining 3 heads and 2 tails?
- (a) $\frac{5}{16}$ (b) $\frac{3}{5}$ (c) $\frac{3}{32}$ (d) $\frac{3}{10}$ (e) $\frac{1}{2}$
6. A scarecrow, a tin man, a lion, a dog, and a little girl stand in random order in a line to meet a wizard. What is the probability that the lion and the dog are standing next to each other?
- (a) $\frac{1}{4}$ (b) $\frac{2}{5}$ (c) $\frac{1}{2}$ (d) $\frac{4}{7}$ (e) $\frac{5}{8}$
7. Mr. Kafka has 5 pairs of gloves. If he randomly picks a right glove and a left glove, what is the probability that the gloves match?
- (a) $\frac{1}{25}$ (b) $\frac{2}{25}$ (c) $\frac{1}{5}$ (d) $\frac{1}{20}$ (e) $\frac{4}{25}$

In the next 2 problems, let S be a sample space with E , F , and G events associated to S . Assume that $\Pr(E) = 0.4$ $\Pr(F') = 0.5$ $\Pr(G') = 0.6$ and $\Pr(F \cap G) = 0.2$.

8. If E and F are independent, what is $\Pr(EIF)$?

- (a) .4 (b) $\frac{4}{5}$ (c) .5 (d) .02 (e) $\frac{4}{9}$

9. Calculate $\Pr(F|G)$.

- (a) .02 (b) .03 (c) $\frac{2}{5}$ (d) $\frac{1}{2}$ (e) .2

10. To win a particular game, a person must roll a die and get a 1 or 2. To start, the person tosses a coin. If it lands heads, then they get only one roll of a die to get a 1 or 2. If it lands tails, then they get two rolls of a die to get a 1 or 2. What is the probability of winning the game?

- (a) $\frac{1}{12}$ (b) $\frac{5}{12}$ (c) $\frac{4}{9}$ (d) $\frac{1}{6}$ (e) $\frac{7}{10}$

11. If the odds in favor of an event are 3 to 5, what is the probability that the event will occur?

(a) $\frac{3}{5}$

(b) $\frac{2}{5}$

(c) $\frac{3}{8}$

(d) $\frac{5}{8}$

(e) $\frac{8}{15}$

12. A pair of dice is rolled. What is the probability that one or more of the top two faces is a 5 given that the sum of the top two faces is 9?

(a) $\frac{1}{6}$

(b) $\frac{5}{36}$

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

(e) $\frac{2}{9}$

13. A bucket contains 4 white spheres and 4 red spheres. 3 spheres are selected at random. What is the probability that at least one of the selected spheres is white?

(a) $\frac{3}{4}$

(b) $\frac{13}{14}$

(c) $\frac{1}{10}$

(d) $\frac{19}{24}$

(e) $\frac{5}{6}$

number of heads

Proportion of
population

Proportion which
are magic

Table for
Problem 14

| | | |
|---|----|----|
| 1 | .2 | .3 |
| 2 | .3 | .8 |
| 3 | .5 | .6 |

14. An island contains one-headed, two-headed, and three-headed dragons. The table above shows the proportion of each in the general dragon population and their probability of being magic. What is the probability that a randomly chosen dragon has 3 heads given that it is magic?

- (a) $\frac{1}{2}$ (b) .6 (c) .3 (d) $\frac{1}{11}$ (e) .18



Histogram for
Problem 15.

15. The histogram above represents a probability distribution. What is the probability that the outcome is greater than or equal to 4?

- (a) .2 (b) .3 (c) .4 (d) .6 (e) .8

In the next 2 problems X is a random variable. Let X have the following probability distribution.

| k | $\Pr(X = k)$ |
|-----|--------------|
| -1 | .1 |
| 0 | .2 |
| 1 | .4 |
| 2 | .3 |

Probability distribution
For problems 16 and 17.

16. What is the expected value of X ?

- (a) 0 (b) 1 (c) .3 (d) .5 (e) .9

17. Find the probability distribution of X^2 .

(a)

| k | $\Pr(X^2 = k)$ |
|-----|----------------|
| 0 | .4 |
| 1 | .5 |
| 2 | .2 |

(b)

| k | $\Pr(X^2 = k)$ |
|-----|----------------|
| 0 | .2 |
| 1 | .5 |
| 4 | .3 |

(c)

| k | $\Pr(X^2 = k)$ |
|-----|----------------|
| -1 | .01 |
| 0 | .04 |
| 1 | .16 |
| 2 | .09 |

(d)

| k | $\Pr(X^2 = k)$ |
|-----|----------------|
| 0 | .04 |
| 1 | .25 |
| 4 | .09 |

(e)

| k | $\Pr(X^2 = k)$ |
|-----|----------------|
| -1 | .1 |
| 0 | .2 |
| 1 | .4 |
| 2 | .3 |

| number of heads | number of occurrences |
|-----------------|-----------------------|
| 0 | 2 |
| 1 | 1 |
| 2 | 4 |
| 3 | 2 |
| 4 | 1 |

Frequency distribution for Problem 18.

18. An experiment consists of flipping a coin 4 times and counting the number of heads. The experiment is repeated 10 times. Above is a frequency distribution table of the outcomes. What is the mean?
- (a) 1.9 (b) 2 (c) 1 (d) .26 (e) 2.5
19. An experiment consists of flipping a coin 2 times and counting the number of tails. Compute the variance of the probability distribution of the experiment.
- (a) 1 (b) $\frac{1}{16}$ (c) $\frac{3}{8}$ (d) $\frac{1}{2}$ (e) $\frac{3}{4}$
20. An experiment consists of flipping a coin either 3 times or until a flip lands "heads", whichever comes first. Let X count the number of flips in a trial. What is the expected value of the random variable X ?
- (a) $\frac{7}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{4}$ (e) 0