Formulas: $F = (1 + i)^n P$ $F = s_{n\rceil i} R$ $P = a_{n\rceil i} R$

$$s_{n\rceil i} = \frac{(1 + i)^n - 1}{i} \qquad a_{n\rceil i} = \frac{(1 + i)^n - 1}{i(1 + i)^n}$$

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1. Let A = {a, b, c}, B = {c, d, e} and the universal set U = {a, b, c, d, e, f}. then $(A \cup B) \cap (A \cap B)' =$

a. {c} b. {a, b, c, d, e} c. {a, b, d, e, f} d. {a, b, d, e} e. {c, f}

2. An employee at a car park has to park a Mercedes Benz, a Jaguar and a Ferrari in three of the ten unoccupied parking spaces at the park. In how many ways can this be done?

a. 120 b. 720 c. 1000 d. 30 e. 840

3. If there are twenty-five different recent release videos currently stocked by a video store, how many different selections of three distinct recent release movies are possible?

a. 3^{25} b. P(25,3) c. C(25,3) d. 25^3 e. 3×25

4. A bag of sweets contains nine lollygobble bliss bombs: three are red, three are green and three are yellow. If three sweets are taken at random from the bag, what is the probability that there is one of each color?

a. $\frac{2}{7}$ b. $\frac{9}{28}$ c. $\frac{21}{56}$ d. $\frac{3}{56}$ e. $\frac{3}{28}$

5. Two fair dice are rolled. What is the probability that at least one of the dice comes up as a six, given that the sum of the numbers on their top faces is ten or greater?

a. $\frac{2}{3}$ b. $\frac{3}{4}$ c. $\frac{4}{5}$ d. $\frac{6}{11}$ e. $\frac{5}{6}$

6. For an electronic device to function properly, each of the 40 transistors it contains must work. If each transistor has a 1% chance of failing within 5 years and failures are independent of one another, what is the probability that the device will fail within 5 years?

a. $1 - (.99)^{40}$ b. $1 - (.01)^{40}$ c. $(.01)^{40}$ d. $(.99)^{40}$ e. $(.99)^{5\times 40}$

7. If E and F are two events satisfying $\Pr(E) = .3$, $\Pr(F) = .4$ and $\Pr(E' \cap F') = .4$, what is $\Pr(F \cap E')$?

a. .1 b. 0 c. .3 d. .2 e. .4

8. An expert dart player throws darts at a dart board, stopping when he has either thrown three darts or hit the bullseye. On each throw, his chance of hitting the bullseye is $\frac{1}{3}$. What is the probability that he will throw at least twice, and hit the bullseye?

a. $\frac{8}{27}$ b. $\frac{4}{27}$ c. $\frac{2}{9}$ d. $\frac{10}{27}$ e. $\frac{2}{3}$

9. An absent-minded male nurse is required to give Miss Smith a pill today. The probability that the nurse will forget to administer the pill is $\frac{3}{4}$. If Miss Smith receives the pill, the probability she will die today is $\frac{1}{5}$, but if she doesn't receive the pill, the probability she will die today is $\frac{2}{3}$. Miss Smith died today. What is the probability that the nurse forgot to administer the pill?

a. $\frac{10}{11}$ b. $\frac{3}{4}$ c. $\frac{11}{20}$ d. $\frac{1}{2}$ e. $\frac{2}{3}$

10. A family of two parents and three children arrange themselves randomly in a line to be photographed. What is the probability that the parents are <u>not</u> standing next to each other?

a. $\frac{3}{10}$ b. $\frac{2}{5}$ c. $\frac{4}{5}$ d. $\frac{1}{5}$ e. $\frac{3}{5}$

11. John and Jill play a dice game. A single die is rolled once. If the number appearing is odd, John gives Jill a number of dollars equal to the number of dots on the top face. If the number appearing on top is even, Jill gives John $3. What are Jill's expected winnings if the game is played once?

a. $2 b. $1 c. 0
d. –$1(i.e. a $1 loss) e. –$2 (i.e. a $2 loss)

12. If a fair die is tossed five times, the probability of obtaining six once or twice is

a. $\binom{5}{1}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^1 + \binom{5}{2}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2$

b. $\binom{5}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^4 + \binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3$

c. $\binom{5}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2 + \binom{5}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1$

d. $\binom{5}{1}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^1 + \binom{5}{2}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2$

e. $\left[\binom{5}{1} + \binom{5}{2}\right]\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^2$

13. If the average life of a certain brand of shirt is 3 years with a standard deviation of 8 months, what percentage of shirts would last for 2 years or less? (Assume that shirtlife is normally distributed).

a. 6.7% b. 9.3% c. 40.1% d. 13.6% e. 86.4%

14. Use the normal approximation to the binomial distribution to estimate the probability of obtaining exactly 50 tails when a fair coin is tossed 100 times.

a. .06 b. .04 c. .05 d. .08 e. .07

15. Calculate the variance of the random variable X with probability distribution shown below.

x	Pr (X = x)
0	$\frac{1}{5}$
5	$\frac{3}{5}$
10	$\frac{1}{5}$

a. 15 b. 5 c. 35 d. 25 e. 10

16. Which of the following is an equation for the line which passes through the point (5,4) and is perpendicular to the line $3x + 2y = 7$?

a. $3x - 2y = 2$ b. $3x - 2y = 7$ c. $2x + 3y = 22$

d. $2x - 3y + 2 = 0$ e. $3x - 2y + 2 = 0$

17. Which of the following statements concerning the solution of the simultaneous equations $\begin{cases} 2x - 3y = 6 \\ 3x - 5y = 8 \end{cases}$ is true?

a. $y = -2$ b. $x = 2$ c. $x = 6$ d. $x = -2$ e. $y = -6$

18. Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \\ 0 & 2 & 3 \end{bmatrix}$. Which of the following is the entry in the first row and third column of A^{-1}?

a. 9 b. -3 c. 5 d. -7 e. -17

19. The augmented matrix of a system of linear equations is

$$\begin{array}{c} \begin{matrix} x & y & z & w \end{matrix} \\ \left[\begin{array}{cccc|c} 1 & -\frac{1}{5} & 0 & 2 & 6 \\ 0 & 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}.$$

Which of the following is the general solution of this system of equations?

a. $\begin{cases} x = 6 + \frac{1}{5}y - 2w \\ z = -4 - 3w \\ y = \text{any value} \\ w = \text{any value} \end{cases}$

b. $\begin{cases} x = -6 + \frac{1}{5}y - 2w \\ z = 4 - 3w \\ y = \text{any value} \\ w = \text{any value} \end{cases}$

c. $\begin{cases} x = 6 - \frac{1}{5}y + 2w \\ z = -4 + 3w \\ y = \text{any value} \\ w = \text{any value} \end{cases}$

d. $\begin{cases} x = -6 - \frac{1}{5}y + 2w \\ z = -4 + 3w \\ y = \text{any value} \\ w = \text{any value} \end{cases}$

e. There is no solution.

20. Which of the following matrices is the value of the matrix product AB, where $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 0 & -1 \\ 1 & 1 \end{bmatrix}$?

a. $\begin{bmatrix} -1 & 1 & 3 \\ 1 & -1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 2 & 4 \\ -1 & -3 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

d. $\begin{bmatrix} 2 & -1 \\ 4 & -3 \end{bmatrix}$ e. $\begin{bmatrix} 1 & -3 \\ 2 & -5 \end{bmatrix}$

21. Let A be the 2×2 matrix such that $A^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 3 \end{bmatrix}$. Then the solution of the matrix equation $A \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$ is

a. $\begin{cases} x = a + 2b \\ y = 3a + 3b \end{cases}$ b. $\begin{cases} x = a + 3b \\ y = 2a + 3b \end{cases}$ c. $\begin{cases} x = a + \frac{2}{3} b \\ y = a - \frac{1}{3} b \end{cases}$

d. $\begin{cases} x = 3a - 2b \\ y = -3a + b \end{cases}$ e. $\begin{cases} x = -a - \frac{2}{3} b \\ y = -a - \frac{1}{3} b \end{cases}$

22. If a tourist spends a night in a hotel, the following night she is equally likely to camp or to spend the night in a hotel. If she camps one night, she is certain to spend the next night in a hotel. In the long run, what proportion of her holiday nights will the tourist spend in a hotel?

a. $\frac{1}{2}$ b. $\frac{3}{4}$ c. $\frac{2}{3}$ d. $\frac{5}{6}$ e. $\frac{7}{8}$

23. Which of the following matrices is an absorbing stochastic matrix (i.e. is both a stochastic matrix and an absorbing matrix)?

a. $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{1}{6} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{2} & 1 \end{bmatrix}$

b. $\begin{bmatrix} 0 & \frac{1}{3} \\ 1 & \frac{2}{3} \end{bmatrix}$

c. $\begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{4} \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ \frac{2}{3} & 0 & 0 & \frac{3}{4} \end{bmatrix}$

d. $\begin{bmatrix} \frac{1}{3} & 1 & 0 \\ \frac{1}{3} & 0 & 0 \\ \frac{1}{3} & 0 & 1 \end{bmatrix}$

e. $\begin{bmatrix} 0 & 1 & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

24. A fly is confined to a locked room. There is a piece of sticky flypaper in the corner of the room. If the fly lands on the flypaper, it stays trapped there. If the fly is currently on the window ledge, then 10 seconds later there is a probability of 0.1 it will still be there, a probability of 0.8 that it will be on a dirty plate on the table, and a probability of 0.1 that it will have been trapped by the flypaper. If its currently on a dirty plate, then 10 seconds later there is a probability of 0.9 it will still be on a dirty plate and of 0.1 that it will be on the window ledge. What is the probability that if the fly is currently on the window ledge, then the flypaper will eventually trap the fly?

a. .3 b. .8 c. .6 d. .5 e. 1

25. Five years ago Allison deposited an amount of money in an account paying 8% annual interest compounded quarterly. Today, the account contains $1,000.00. If Allison has not used the account since the initial deposit, how much was her initial deposit?

a. $905.29 b. $672.97 c. $747.26 d. $943.40 e. $609.53

26. How much money must Betty deposit now into a savings account earning 6% annual interest compounded monthly in order to be able to withdraw $1,000 at the end of each month for five years.

a. $14,332.80 b. $42,123.64 c. $55,498.45

d. $51,725.56 e. $69,770.03

27. Claudette invests $10,000 in an account paying 12% annual interest compounded semiannually. How much will she have at the end of 30 years?

a. $517,255.61 b. $32,987.69 c. $53,312.82

d. $51,725.56 e. $329,876.91

28. At the end of every month Dianne deposits $200 into a savings account receiving 6% annual interest compounded monthly. How much will Dianne have in the account at the end of 10 years?

a. $26,361.59 b. $327,758.69 c. $2,636.16

d. $32,775.87 e. $18,014.69

29. Eve would like to buy a $20,000 yacht when she retires in 15 years. How much should she deposit at the end of each month into an account yielding 8% annual interest compounded quarterly so that she will have enough money to buy the yacht?

a. $228.10 b. $175.36 c. $232.76

d. $287.68 e. $205.92

30. Felicity has taken out a 30-year $100,000 mortgage at 6% annual interest compounded quarterly. What is the quarterly payment?

a. $1801.85 b. $726.49 c. $554.98

d. $3312.88 e. $790.58