Formulas: $F = (1 + i)^n P$ $F = s_{n\rceil i} R$ $P = a_{n\rceil i} R$

$$s_{n\rceil i} = \frac{(1 + i)^n - 1}{i} \qquad a_{n\rceil i} = \frac{(1 + i)^n - 1}{i(1 + i)^n}$$

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1. In a group of people, 35 know how to snow ski, and 25 know how to water ski. If 30 know either how to snow ski or water ski but not both, how many people in the group can both snow ski and water ski?

a. 10 b. 20 c. 15 d. 5 e. 25

2. If A and B are subsets of a universal set U and $A' \cup B = U$, which of the following **must** be true?

a. B is a subset of A b. A is a subset of B c. $A = \varnothing$

d. $B = U$ e. $A \cap B = \varnothing$

3. Let A, B, C be three events in a sample space S such that

   $Pr(A) = .28$ $Pr(B) = .51$ $Pr(C) = .50$

   $Pr(A \cap B) = .14$ $Pr(B \cap C) .19$ $Pr(A \cap C) = .10$

   and $Pr(A \cap B \cap C) = .04$?

   What is $Pr(A \cup B \cup C)$?

a. .83 b. .84 c. .81 d. .87 e. .90

4. What is the coefficient of $x^3 y^5$ in $(x + y)^8$?

a. 56 b. 54 c. 45 d. 84 e. 48

5. Five children are to be lined up in a row for a photograph. In how many ways can this be done if two of the children (Mary and John) refuse to stand next to each other?

a. 60 b. 72 c. 120 d. 48 e. 65

6. A chemistry lab class containing 20 students is to split into 10 groups of 2 for an experiment. In how many ways can this be done?

a. $\frac{20!}{2^{20} \cdot 10!}$ b. $\frac{20!}{2^{10}}$ c. $\frac{20!}{(10!)^2}$ d. $\frac{20!}{2^{10} \cdot 10!}$ e. $\frac{20!}{2^{10} \cdot (10!)^2}$

7. An urn contains 2 red balls, 2 white balls and 2 green balls. Three balls are drawn without replacement from the urn. What is the probability that the balls are all of different colors?

a. $\frac{2}{3}$ b. $\frac{1}{5}$ c. $\frac{1}{3}$ d. $\frac{1}{2}$ e. $\frac{2}{5}$

8. A fair die is rolled three times. What are the **odds in favor** of obtaining different numbers on the top face in all three rolls?

a. 4:1 b. 4:5 c. 5:4 d. 9:4 e. 4:9

9. What is the probability that a hand of five cards dealt from a standard deck of 52 will contain four cards of one **denomination** (i.e. face value) and one other card?

a. $\frac{13 \cdot 48}{C(52,5)}$ b. $\frac{C(13,4) \cdot 39}{C(52,5)}$ c. $\frac{13 \cdot 48}{P(52,5)}$ d. $\frac{P(13,4) \cdot 39}{P(52,5)}$ e. $\frac{13 \cdot 4 \cdot 48}{C(52,5)}$

10. A carton of 12 eggs on a supermarket shelf contains two eggs with cracks on the bottom, not visible unless they are removed. To check for broken eggs, a shopper removes three randomly chosen eggs from the carton and examines them. What is the probability the customer will find at least one of the broken eggs in the carton this way?

a. $\frac{2}{11}$ b. $\frac{5}{11}$ c. $\frac{1}{6}$ d. $\frac{2}{7}$ e. $\frac{3}{8}$

11. A fair coin is tossed four times. Find the probability of obtaining four heads, given that at least three heads were obtained. a. $\frac{1}{7}$ b. $\frac{1}{16}$ c. $\frac{5}{16}$ d. $\frac{1}{4}$ e. $\frac{1}{5}$

12. An urn contains two white balls, a red ball and a green ball. Balls are drawn one at a time from the urn without replacement, till none are left. Find the probability that the two white balls are drawn before the red ball.

a. $\frac{1}{6}$ b. $\frac{5}{24}$ c. $\frac{1}{3}$ d. $\frac{1}{4}$ e. $\frac{7}{24}$

13. An electrical circuit contains 10 components, which function independently of one another. The probability that a component will fail within 5 years is 0.01. For the circuit to function correctly, all 10 components must work. What is the probability that the circuit will function correctly for five years?

a. $1 - (0.01)^{10}$ b. $(.99)^{10}$ c. $1 - (.99)^{10}$ d. $(.01)^{10}$ e. .1

14. A screw manufacturer has plants in Pittsburgh, Detroit and Chicago. 30% of its screws are manufactured in Pittsburgh, and 1% of those are defective. 40% are manufactured in Detroit, and 2% of those are defective. The remainder of the screws are made in Chicago, and 0.5% of these are defective. If a screw is defective, what is the probability it was made in Pittsburgh.

a. $\frac{6}{25}$ b. $\frac{2}{13}$ c. $\frac{3}{13}$ d. $\frac{1}{10}$ e. $\frac{1}{5}$

15. The following table is the probability distribution table of a random variable X.

k	Pr(X = k)
0	.1
1	.2
2	.3
3	a
4	b

Which of the following is a possible pair of values for a and b compatible with this information?

a. a = .4, b = .5 b. a = .1, b = .3 c. a = .2, b = .3

d. a = .1, b = .5 e. a = .1, b = .2

16. At a mini-golf course, the probability distribution for the number of shots X required on the opening hole is a follows

k	Pr(X = k)
2	$\frac{2}{7}$
3	$\frac{3}{7}$
4	$\frac{2}{7}$

What is the variance of X?

a. $\sqrt{\frac{3}{7}}$ b. $\sqrt{\frac{4}{7}}$ c. $\frac{3}{7}$ d. $\frac{4}{7}$ e. $\sqrt{\frac{2}{7}}$

17. Tom and Mary play a game as follows. A die is rolled; if it shows 6 on the top face, Mary gives Tom $6, otherwise Tom gives Mary $1. What would be the expected value of Tom's winnings if the game were to be played 12 times?

a. $0 b. $1 c. $2 d. – $1 e. – $2

18. The lengths of newly hatched alligators are normally distributed with a mean of 9cm and a standard deviation of 1cm. What percentage of the time a newly hatched alligator will be less than 7.5 cm long?

a. 6.7% b. 8.5% c. 4.2% d. 2.4% e. 1.0%

19. A single fair die is rolled 180 times. Use the normal approximation to the binomial distribution to estimate what percentage of the time the top face shows a six 35 or more times.

a. 22% b. 16% c. 20% d. 18% e. 24%

20. A multiple choice test has 25 questions, each with 5 possible answers. A student is able to answer 15 of these questions correctly, but must guess the remaining answers. What is the probability the student will end up with 20 correct answers out of the 25?

a. $C(15,10)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^{10}$ b. $C(25,20)\left(\frac{1}{5}\right)^{20}\left(\frac{4}{5}\right)^5$ c. $C(25,20)\left(\frac{1}{5}\right)^5$

d. $C(15,10)\left(\frac{4}{5}\right)^{20}\left(\frac{1}{5}\right)^{10}\left(\frac{4}{5}\right)^5$ e. $C(10,5)\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^5$

21. At a certain fossil site in Montana, fossils dated at 60 million years are found at a depth of 115' from the top of an exposed cliff face, and fossils dated at 85 million years are found at a depth of 175'. Assuming that fossil age and depth are related by a linear (straight line) equation, at what depth should a paleontologist search for fossils from the time of extinction of the dinosaurs (65 million years ago)?

a. 127' b. 135' c. 142' d. 145' e. 120'

22. Which of the following is the entry in the second row and third column of the matrix C, where $C = \begin{bmatrix} 1 & -4 & 5 \\ -2 & 1 & 3 \\ 6 & -3 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & -3 \\ -3 & -1 & 5 \\ 5 & 0 & -2 \end{bmatrix}$

a. –1 b. 17 c. 5 d. 27 e. 21

23. Consider the system of equations

$$\begin{cases} 2x + 4y + 7z + 9w = 4 \\ x + 2y + 3z + 4w = 1 \\ 3x + 6y + 4z + 7w = -7 \end{cases}.$$

Which of the following is the correct general solution of this system of equations?

a. $\begin{cases} x = -3 - w \\ y = -1 \\ z = 2 - w \\ w = \text{any number} \end{cases}$ b. $\begin{cases} x = -5 - 2y - w \\ y = \text{any number} \\ z = 2 - w \\ w = \text{any number} \end{cases}$ c. $\begin{cases} x = -5 - 2y - w + 3z \\ y = \text{any number} \\ z = \text{any number} \\ w = \text{any number} \end{cases}$

d. $\begin{cases} x = -5 + 2y - 2w \\ y = \text{any number} \\ z = 2 + w \\ w = \text{any number} \end{cases}$ e. $\begin{cases} x = -3 - 2w \\ y = -1 \\ z = 2 - w \\ w = \text{any number} \end{cases}$

24. Which of the following is the solution for x, y of the matrix equation

$$\begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}.$$

a. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -7 & -2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$ b. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -4 & 7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

c. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$ d. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 4 & -7 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

e. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$

25. Which of the following is the entry in the second row and third column of the matrix A, where

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 5 & -2 \\ 0 & 0 & 2 \end{bmatrix}^{-1}.$$

a. 7 b. 0 c. −5 d. 5 e. −7

26. Consider the following matrices

$$X = \begin{bmatrix} \frac{1}{2} & 1 \\ \frac{1}{2} & 0 \end{bmatrix} \qquad Y = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{6} & \frac{3}{4} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{2} \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{bmatrix}$$

Which of these matrices are regular stochastic matrices?

a. X only b. Y only c. Z only

d. X and Y only e. X and Z only

27. A penguin has two ways of spending a day, either on shore in its rookery or at sea, feeding on fish. If the penguin stays in the rookery one day, there is a 90% chance it will go to sea the following day, whereas if it feeds one day, there's only a 70% chance it will feed the next day also. In the long run, what percentage of days does the penguin spend feeding at sea?

a. 85% b. 80% c. 75% d. 83% e. 78%

28. Dan invested $10,000 at 12% annual interest compounded **monthly**. How much will he have at the end of 15 years?

a) 16,678.34 b) 59,958.02 c) 49,958.02

d) 83,321.66 e) 34,548.15

29. Sue took out a 25-year $60,000 mortgage at 6% annual interest compounded **monthly**. What is the monthly payment?

a) $144.30 b) $155.21 c) $386.58 d) $692.99 e) $865.81

30. At the end of every 3 months Jason deposits $100 into a savings account receiving 6% annual interest compounded **quarterly**. How much will Jason have in the account at the end of 5 years?

a) 5,331.28 b) 6,977.00 c) 4,324.46 d) 1,716.86

e) 2,312.37

31. How much money must you deposit now into a savings account receiving 8% annual interest rate compounded **quarterly** in order to be able to withdraw $2000 at the end of each quarter year for 12 years?

a) 61,346.24 b) 68,085.11 c) 158,707.01 d) 50,977.68

e) 69,521.77

32. George took out a loan in the amount of $563. He paid off the loan in 5 months with monthly payments of $116. How much interest did he pay?

a) $56.30 b) $116 c) $11.60 d) $44.7 e) $17

33. Helen would like to buy a $50,000 recreational vehicle when she retires in 10 years. How much should she deposit at the end of each month into an account receiving 12% annual interest compounded **monthly** so that she will have enough money to purchase the vehicle?

a) $348.50 b) $717.35 c) $555.10 d) $217.35 e) $331.29

34. Find the stable distribution $\begin{bmatrix} x \\ y \end{bmatrix}$ of the matrix $\begin{bmatrix} .2 & .4 \\ .8 & .6 \end{bmatrix}$.

a) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$ b) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$ c) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$

d) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$ e) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{1}{4} \\ \frac{1}{4} \end{bmatrix}$

35. If $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$ is the transition matrix of a Markov process and the initial distribution is $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Then the next two distributions are:

a) $\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$ and $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$ b) $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$ and $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$ c) $\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

d) $\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$ and $\begin{bmatrix} \frac{5}{16} \\ \frac{11}{16} \end{bmatrix}$ e) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$