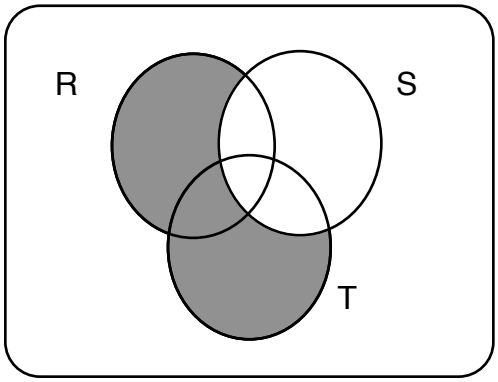
MATH 104 - FINAL EXAM

1. Which of the following sets describes the shaded area of the Venn diagram:



(a) $(R \cup T) \cap S'$ (b) $(R \cap T) \cup S'$ (c) $R \cup (T \cap S')$

(d) $(R \cap T) \cap S'$ (e) $(R \cup T) \cup S'$

2. Out of 50 students who exercise regularly, 25 jog, 20 play basketball and 15 swim. 10 play basketball and jog, 5 play basketball and swim, 7 jog and swim and 2 people do all three. How many students do not do any of these activities?

(a) 4 (b) 15 (c) 0 (d) 10 (e) 2

3. A poker hand consists of five cards selected at random from a deck of 52. How many different poker hands consist entirely of Kings and Queens?

(a) $\frac{C(8,5)}{P(8,5)}$ (b) $P(8,5)$ (c) $C(4,2)\,C(4,3)$ (d) $C(52,5) - C(8,5)$

(e) $C(8,5)$

4. In how many ways can a teacher divide a class of 30 students into 6 groups of 5.

(a) $\frac{30}{(5!)^6}$ (b) $\frac{30!}{5!\,(6!)^5}$ (c) $\frac{30!}{6!\,(5!)^6}$

(d) $\frac{30!}{5!\,6!}$ (e) $\frac{30!}{(6!)^5}$

5. In how many ways can 5 boys and 6 girls be lined up for a photograph so that no two girls are adjacent.

(a) 11! - 6! (b) 6! 5! (c) P(11,6) P(11,5) - 6!

(d) C(11,6) 6! 5! (e) C(11,6) C(11,5) - 6!

6. An experiment consists of drawing a letter at random from the collection {w, e, l, u, v, x, a, m, s}. Let E be the event "A vowel is drawn" and F the event "an m or an s is drawn". Then $F \cup E'$ corresponds to the event,

(a) {m, s, w, l, v, x} (b) {m, s} (c) {e, u, a, m, s} (d) $\varnothing$

(e) {w, l, v, x}.

7. An experiment consists of drawing 3 balls at random from an urn containing 2 red balls and 4 white balls. What is the probability of getting at least 2 white balls?

(a) $\frac{2}{3}$ (b) $\frac{3}{5}$ (c) $\frac{2}{5}$ (d) $\frac{1}{5}$ (e) $\frac{4}{5}$

8. Let E and F be events where $\Pr(E') = \frac{1}{4}$, $\Pr(F) = \frac{1}{4}$ and $\Pr(E \cap F) = \frac{1}{8}$. Find $\Pr(E \cup F)$.

(a) $\frac{5}{8}$ (b) 1 (c) $\frac{7}{8}$ (d) $\frac{1}{2}$ (e) $\frac{3}{8}$

9. A new piece of electronic equipment has five components. The probability of failure within a year is .1 for each component. Assuming that the failure of the various components are independent of each other. What is the probability that no component will fail in the 1st year?

(a) .1 (b) $(.1)^5$ (c) $1 - (.1)^5$ (d) $1 - (.9)^5$ (e) $(.9)^5$

10. John, James and Joseph are responsible for the output of decanters at a glass factory. The table below shows the proportion of the output for which each is responsible and the probability that a decanter chosen at random from their respective outputs is defective. If a decanter shipped to Notre Dame bookstore is defective, what is the probability that it was produced by John?

Worker	Proportion of Output	Pr(Defective)
John	.5	.1
James	.3	.5
Joseph	.2	.7

(a) $\frac{5}{29}$ (b) $\frac{5}{34}$ (c) .05 (d) .5 (e) .1

11. A fair die is tossed 5 times. What is the probability of getting at least two threes?

(a) $1 - \left(\frac{5}{6}\right)^5 - 5\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$ (b) $\left(\frac{1}{6}\right)^2$

(c) $\left(\frac{5}{6}\right)^5 + C(5,1)\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + C(5,2)\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^2$

(d) $1 - 5\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$ (e) $C(5,2)\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^2$

12. Abe and Beryl play a game. They throw a die. If the outcome is a six, Abe gives Beryl $1 and if the outcome is not a six Beryl gives Abe $2. What are the expected winnings for Abe in this game?

(a) $1.00 (b) $3.00 (c) Approx. $1.67 (d) $1.50 (e) $2.00

13. The probability distribution for a random variable X is given below. Find the variance, σ^2, of X.

k	Pr(X=k)
2	$\frac{1}{8}$
3	$\frac{1}{8}$
4	$\frac{1}{2}$
5	$\frac{1}{8}$
6	$\frac{1}{8}$

(a) 0 (b) $\sqrt{\frac{7}{4}}$ (c) $\sqrt{\frac{5}{4}}$ (d) $\frac{5}{4}$ (e) $\frac{7}{4}$

14. Find the area under the standard normal curve between z = -2 and z = +3.

(a) .0241 (b) .9759 (c) .9785 (d) .9987 (e) .9772

15. A squash racquet manufacturer finds that 10% of the squash racquets he produces need to be restrung within 1 year of leaving the factory. Using the normal approximation to the binomial, find the probability that 15 or more out of a shipment of 100 will have to be restrung within 1 year of sale.

(a) .9999 (b) .0495 (c) .0668 (d) .9505 (e) .9032

16. Two long distance telephone companies offer calling plans as follows for international calls. The first company charges $15.00 per month plus $0.47 per minute for calls to England. The second company charges $10.00 per month plus $0.52 per minute for calls to England. For what monthly length of time spent calling England would the two plans charge the same price?

(a) 1 hr. 20 min. (b) 1 hr. 40 min. (c) 1 hr. 30 min. (d) 1 hr. 10 min. (e) 1 hr.

17. A truck carrying citrus fruit contains a total of 100 crates of fruit, of which some crates contain oranges, some contain limes and some contain lemons. A crate of oranges weighs 20 lbs. and costs $40.00, a crate of limes weighs 10 lbs. and costs $30.00, and a crate of lemons weighs 15 lbs. and costs $20.00. If the total weight of the crates is 1700 lbs. and the total value of the crates is $3400, how many crates of oranges are there?

(a) 60 (b) 70 (c) 50 (d) 55 (e) 65

18. Given that $A^{-1} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, which of the following statements is true about the solution of the matrix equation $A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$?

(a) x = - 2 (b) z = 1 (c) x = 2 (d) y = 1 (e) y = - 4

19. Which of the following is the entry in the first row and third column of A^{-1}, where A denotes the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(a) - 2 (b) - 1 (c) 1 (d) 2 (e) 0

20. Which of the following statements is true about the system of equations

$$\begin{cases} x + y + z = 5 \\ x + 2y + 3z = 8 \\ 3x + 4y + 5z = 12 \end{cases} ?$$

(a) There is no solution (b) $x = 1$ (c) $z = 3$

(d) $y = 2$ (e) There are infinitely many solutions

21. The matrix of a system of equations is reduced to the form

$$\left[\begin{array}{cccc|c} x & y & z & u & \\ 1 & 2 & 0 & 4 & 1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Which of the following represents the general solution of this system?

(a) $\begin{cases} x = 1 \\ y = 2 \\ z = \text{any number} \\ u = 4 - 3z \end{cases}$ (b) $\begin{cases} x = 1 - 2y + 4u \\ y = \text{any number} \\ z = 2 - 3u \\ u = \text{any number} \end{cases}$

(c) $\begin{cases} x = -1 + 2y + 4u \\ y = \text{any number} \\ z = -2 + 3u \\ u = \text{any number} \end{cases}$ (d) $\begin{cases} x = 1 - 2y - 4u \\ y = \text{any number} \\ z = 2 - 3u \\ u = \text{any number} \end{cases}$

(e) There is no solution.

22. If someone has a cold, there is a chance of $\frac{2}{3}$ they will still have the cold next week, while if they do not have a cold, there is a chance of only $\frac{1}{4}$ they will have a cold next week. Using C (cold) and N (no cold) to denote possible states, which of the following is a transition matrix describing this situation?

(a)
$$\begin{array}{c} \\ C \\ N \end{array}\begin{array}{c} \begin{array}{cc} C & N \end{array} \\ \begin{bmatrix} 2/3 & 1/4 \\ 1/3 & 3/4 \end{bmatrix} \end{array}$$

(b)
$$\begin{array}{c} \\ C \\ N \end{array}\begin{array}{c} \begin{array}{cc} C & N \end{array} \\ \begin{bmatrix} 2/3 & 3/4 \\ 1/3 & 1/4 \end{bmatrix} \end{array}$$

(c)
$$\begin{array}{c} \\ C \\ N \end{array}\begin{array}{c} \begin{array}{cc} C & N \end{array} \\ \begin{bmatrix} 2/3 & 1/3 \\ 1/4 & 3/4 \end{bmatrix} \end{array}$$

(d)
$$\begin{array}{c} \\ C \\ N \end{array}\begin{array}{c} \begin{array}{cc} C & N \end{array} \\ \begin{bmatrix} 2/3 & 1/3 \\ 3/4 & 1/4 \end{bmatrix} \end{array}$$

(e)
$$\begin{array}{c} \\ C \\ N \end{array}\begin{array}{c} \begin{array}{cc} C & N \end{array} \\ \begin{bmatrix} 1/3 & 3/4 \\ 2/3 & 1/4 \end{bmatrix} \end{array}$$

23. Which of the following is a regular stochastic matrix?

(a) $\begin{bmatrix} -1/2 & 1 & 1/2 \\ 1 & -1/2 & 1 \\ 1/2 & 1/2 & -1/2 \end{bmatrix}$ (b) $\begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/4 & 1/2 & 1/4 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$ (e) $\begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$

24. The equation of the line through the point (1,2) perpendicular to the line x = 2 is

(a) x = 2 (b) y = 1 (c) y = 2 (d) x = 1 (e) there is no such line

25. Eric, Eva and Emmanuel throw a frisbee amongst themselves. Eric and Eva are equally likely to throw to either of the two others, but Emmanuel always throws to Eric. In the long run, what proportion of the time will Eric have the frisbee?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{4}{9}$ (d) $\frac{5}{9}$ (e) $\frac{2}{9}$

26. An absorbing Markov process has transition matrix

$$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}.$$

If the initial distribution is $\begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$, which of the following is the long run probability of being in State I?

(a) $\frac{1}{4}$ (b) $\frac{2}{9}$ (c) $\frac{5}{9}$ (d) 1 (e) $\frac{4}{9}$

27. Paula invests $2000.00 in an account earning 8% interest p.a. compounded quarterly. How much will her savings have grown to after 5 years?

(a) $2971.89 (b) $4021.94 (c) $3281.03 (d) $2621.46

(e) $3300.38

28. Quentin is saving money to be able to buy a house in 5 years. How much must he deposit each month in an account earning 6% p.a. interest compounded monthly so he will have saved $20,000.00 for a deposit at the end of the five years?

(a) $312.70 (b) $423.62 (c) $307.85 (d) $286.66 (e) $354.79

29. Ralph has won $50,000.00 in a lottery, and invests it in an account earning 12% p.a. interest compounded semiannually. How much money should he withdraw from the account every 6 months if he plans to completely use the winnings and accumulated interest in 10 years and leave the account empty?

(a) $4359.23 (b) $2891.46 (c) $2161.47 (d) $2352.89

(e) $2028.14

30. Sasha has $30,000.00 deposit to put towards the purchase of a $130,000.00 condo. The balance is financed at 9% interest compounded monthly for 30 years. What is the monthly payment?

(a) $467.28 (b) $859.17(c) $804.62 (d) $1124.01 (e) $793.17