MATH 104 FINAL 2nd draft(4-30-96)

FORMULAS:

$F = (1 + i)^n P$, $F = s_{\overline{n|}\,i} R$, $P = a_{\overline{n|}\,i} R$

$A = (1 + nr) P$

$\begin{bmatrix} I & S \\ 0 & R \end{bmatrix}$ $\begin{bmatrix} I & S(I-R)^{-1} \\ 0 & 0 \end{bmatrix}$

$\sigma^2 = (x_1 - \mu)^2 p_1 + (x_2 - \mu)^2 p_2 + \cdots + (x_n - \mu)^2 p_n$

$\sigma^2 = E(X^2) - \mu^2$

$\Pr(X = k) = \binom{n}{k} p^k q^{n-k}$, $\mu = np$, $\sigma = \sqrt{npq}$

$\Pr(B_1|A) = \dfrac{\Pr(B_1)\Pr(A|B_1)}{\Pr(B_1)\Pr(A|B_1) + \Pr(B_2)\Pr(A|B_2)}$)

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1. Consider the following sets.

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$A = \{2, 4, 6, 8\}$
$B = \{1, 2, 3, 5, 7\}$

Which of the following statements is true?

a. $A \cap B = \varnothing$
b. $A \cap B = U$
c. $A \cap B$ is a subset of A
d. A is a subset of $A \cap B$
e. none of the above

2. Suppose $n(U) = 100$, $n(A) = 24$, $n(B) = 60$ and $n(A \cap B) = 10$. Then $n(A' \cap B') = ?$

a. 6 b. 26 c. 90 d. 100

e. none of the above

3. How many three-letter words (including nonsense words) can be formed using the letters a, b, c, d, e, f, g, h, i, j if the last letter cannot be a or b and repetition of letters is allowed?

a. 504 b. 512 c. 720 d. 800

e. none of the above

4. A coin is tossed 5 times and the sequence of heads and tails is observed.

The number of different outcomes having more heads than tails is

a. 3 b. 10 c. 12 d. 21

e. none of the above

5. The probability of getting either a black card or an ace in one draw from an ordinary deck of 52 cards is

a. $\frac{26}{52}$ b. $\frac{28}{52}$ c. $\frac{29}{52}$ d. $\frac{30}{52}$

e. none of the above

6. Let E and F be events such that Pr(F) = 0.4 and Pr(E ∩ F) = 0.3.
Then Pr (E′ ∩ F) = ?

a. 0.006 b. 0.05 c. 0.1 d. 0.55

e. none of the above

7. An urn contains 3 white balls and 4 red balls. Two balls are chosen at random. What is the probability that at least one of the balls is red?

a. $\frac{18}{21}$ b. $\frac{12}{21}$ c. $\frac{6}{21}$ d. $\frac{9}{21}$

e. none of the above

8. The probabilities that two species will become extinct in 5 years are 0.3 and 0.2 respectively. Given that these events are independent, what is the probability that at least one group will become extinct in the next 5 years?

a. 0.5 b. 0.06 c. 0.44 d. 0.56

e. none of the above

9. A basketball player makes 60% of all free throws that he tries. What is the probability that, in two free throws, he makes at least one?

a. 0.36 b. 0.6 c. 0.4 d. 0.84

e. none of the above

10. Urn 1 contains 3 red balls and 1 white ball. Urn 2 contains 2 red balls and 2 white balls. An urn is selected at random, and a ball is chosen from the urn. If the ball is red, what is the probability that Urn 1 was chosen?

a. $\frac{3}{5}$ b. $\frac{5}{8}$ c. $\frac{3}{8}$ d. $\frac{3}{4}$

e. none of the above

11. A single die is tossed 6 times. The probability that a 2 appears exactly 4 times is

a. $\frac{25}{6^6}$ b. $\frac{375}{6^6}$ c. $\frac{15}{6^4}$ d. $\frac{2}{3}$

e. none of the above

12. Consider the probability distribution below.

k	Pr(X = k)
-2	.1
0	.2
1	.1
2	.2
3	.4

The mean is

a. .8 b. 1 c. 1.5 d. 3

e. none of the above

13. Consider the probability distribution below.

k	Pr(X = k)
-2	.1
0	.2
1	.1
2	.2
3	.4

The variance is

a. 0 b. 1.18 c. 2.65 d. 3.05

e. none of the above

14. If Z is the standard normal random variable and Pr $(-z \le Z \le z) = .6578$, then z is

a. .4 b. .45 c. .95 d. 1

e. none of the above

15. A true-false exam consists of 100 questions. What is the probability that someone guessing will get no more than 50 correct answers?

a. 0.4602 b. 0.5398 c. 0.5 d. 0.5793

e. none of the above

16. Suppose X is a normal random variable with mean 12 and standard deviation $\frac{5}{4}$.
A standard value of z = 2 corresponds to an x– value of

a. 9.5 b. 10.4 c. 13.6 d. 14.5

e. none of the above

17. The y-intercept of the line passing through the point (14, 12) and having slope $\frac{2}{7}$ is

a. (0, −4) b. $\left(0, \frac{24}{7}\right)$ c. (0, 8) d. (0, 12)

e. none of the above

18. Consider the system $\begin{cases} x - y = 7 \\ 2x - 2y = k \end{cases}$

Which of the following statements is true?

a. If k = 14, the system has no solution.
b. If k = 14, the system has infinitely many solutions.
c. if k = 14, the system has exactly one solution.
d. If k ≠ 14, the system has exactly one solution.
e. none of the above

19. The identity matrix of size 3 is

a. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

e. none of the above

20. The solution of the matrix equation

$$\begin{bmatrix} 13 & -5 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

is

a. $\begin{cases} x = 3 \\ y = -1 \end{cases}$

b. $\begin{cases} x = 12 \\ y = 31 \end{cases}$

c. $\begin{cases} x = -8 \\ y = 21 \end{cases}$

d. $\begin{cases} x = 23 \\ y = 9 \end{cases}$

e. none of the above

21. Use the Gauss-Jordan method to find the inverse of the matrix

$$A = \begin{bmatrix} -1 & 2 & -4 \\ 1 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

a. $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ -2 & 1 & 1 \end{bmatrix}$ c. $\begin{bmatrix} -1 & 0 & 1 \\ 1 & 2 & 1 \\ 1 & 0 & -2 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ e. none of the above

22. Assume that college graduates and non-college graduates have children in the same numbers. Suppose also that 70% of the children of college-graduates also graduate from college. Of the children of non-college graduates, 55% will graduate from college.

The transition matrix is

a. $\begin{array}{c c} & \begin{array}{cc} G & NG \end{array} \\ \begin{array}{c} G \\ NG \end{array} & \begin{bmatrix} .7 & .3 \\ .55 & .45 \end{bmatrix} \end{array}$ b. $\begin{array}{c c} & \begin{array}{cc} G & NG \end{array} \\ \begin{array}{c} G \\ NG \end{array} & \begin{bmatrix} .3 & .55 \\ .7 & .45 \end{bmatrix} \end{array}$

c. $\begin{array}{c c} & \begin{array}{cc} G & NG \end{array} \\ \begin{array}{c} G \\ NG \end{array} & \begin{bmatrix} .7 & .55 \\ .3 & .45 \end{bmatrix} \end{array}$ d. $\begin{array}{c c} & \begin{array}{cc} G & NG \end{array} \\ \begin{array}{c} G \\ NG \end{array} & \begin{bmatrix} .3 & .45 \\ .7 & .55 \end{bmatrix} \end{array}$

e. none of the above

23. Which of the following are regular stochastic matrices:

I. $\begin{bmatrix} .4 & .5 \\ .6 & .2 \end{bmatrix}$ II. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ III. $\begin{bmatrix} .2 & .3 \\ .1 & .4 \\ .7 & .3 \end{bmatrix}$

IV. $\begin{bmatrix} .1 & .8 \\ .9 & .2 \end{bmatrix}$ V. $\begin{bmatrix} 0 & .6 \\ 1 & .4 \end{bmatrix}$

a. IV only b. I and IV only c. II and III only

d. IV and V only e. II, III and V only

24. The stable distribution for the regular stochastic matrix $\begin{bmatrix} .4 & .2 \\ .6 & .8 \end{bmatrix}$ is

a. $\begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$ b. $\begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$ c. $\begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$

d. $\begin{bmatrix} 3/4 & 3/4 \\ 1/4 & 1/4 \end{bmatrix}$ e. none of the above

25. Consider the matrices:

I. $\begin{bmatrix} 0 & .2 & 0 \\ 1 & .7 & 0 \\ 0 & .1 & 1 \end{bmatrix}$ II. $\begin{bmatrix} 0 & 0 & .3 \\ 1 & 0 & .2 \\ 0 & 1 & .5 \end{bmatrix}$ III. $\begin{bmatrix} .3 & 0 & .5 & 0 \\ .2 & .3 & .5 & 0 \\ .1 & .7 & 0 & 0 \\ .4 & 0 & 0 & 1 \end{bmatrix}$

IV. $\begin{bmatrix} .3 & 1 \\ .4 & 0 \end{bmatrix}$ V. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & .4 & 0 & .3 \\ 0 & 0 & 1 & 0 \\ 0 & .6 & 0 & .7 \end{bmatrix}$

Which ones are absorbing stochastic matrices?

a. I and II only b. I and III only c. II and V only

d. III and V only e. I, II, III, IV and V

26. The stable matrix for the absorbing stochastic matrix $\begin{bmatrix} 1 & 0.4 & 0.2 \\ 0 & 0.1 & 0.2 \\ 0 & 0.5 & 0.6 \end{bmatrix}$ is

a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

e. none of the above.

27. What is the compound amount after 2 years of $100 deposited at 10% interest compounded annually?

a. $121 b. $120 c. $121.55 d. $112

e. none of the above

28. Calculate the amount after 5 years if $2000 is deposited at 13% simple interest.

a. $3684.87 b. $3754.27 c. $3300.00

d. $3791.68 e. none of the above

29. If you deposit $1000 into a fund paying 18% interest compounded monthly, how much can you withdraw at the end of each month for 1 year?

a. $91.68 b. $76.68 c. $86.10

d. $63.81 e. none of the above

30. Calculate the future value of an annuity of $200 per year for 10 years at 6% interest compounded yearly.

a. 2636.16 b. 1517.36 c. 1472.02

d. 3581.68 e. none of the above