1. Let **U** = {a, b, c, d, e, f, g, h, i, j} and let
R = {a, c, e, g, i} ; **S** = {b, c, d, e, f, g} ; **T** = {a, c, d, f, h, i}

Which of the sets below is **(R ∩ T') ∩ S** ?

(a) {c } (b) {a, c, e, f } (c) {d, f } (d) {e, g }

(e) ∅

2. Consider the following set:

U = {all professors}

A = {female professors}

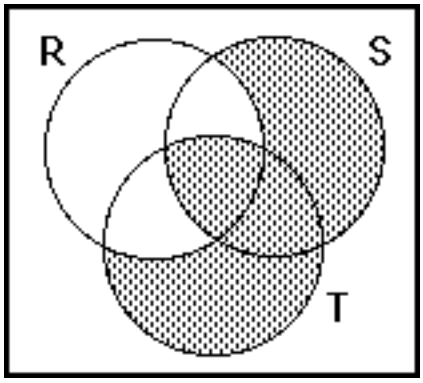
B = {professors under 40 years of age}

(A ∩ B)' is the set

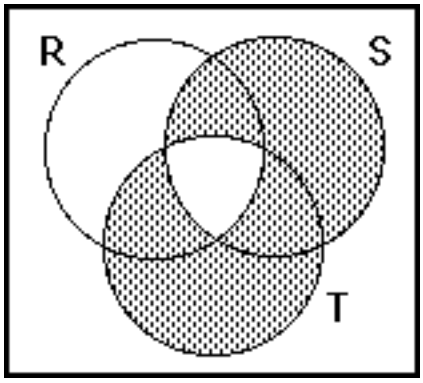
(a) {professors who are male or at least 40 years of age}
(b) {male professors who are at least 40 years of age}
(c) {professors who are male and under 40 years of age}
(d) {professors who are female or under 40 years of age}
(e) none of the above

3. In which Venn diagram does the shaded portion represent **R' ∩ (S ∪ T)** ?

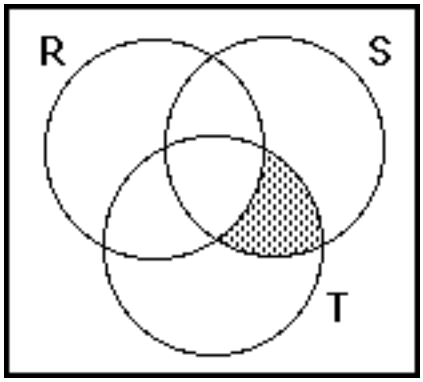
(a)



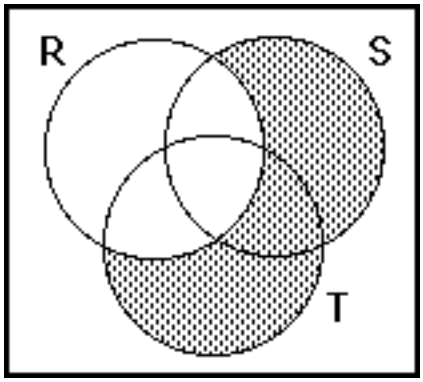
(b)



(c)



(d)



(e) none of the above

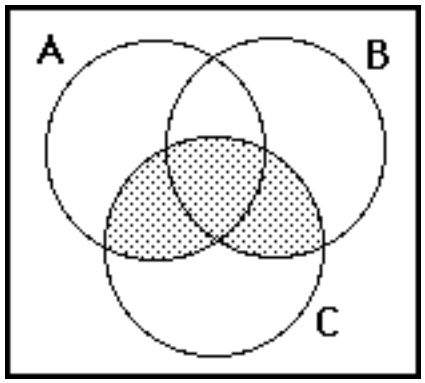
4. A survey of 100 bank customers revealed that 58 of them had a savings account, 63 of them had a checking account, 22 of them had a savings account and a loan, 16 of them had a checking account and a loan, 27 of them have only a checking account, 12 of them had a checking account, a savings account and a loan. Each customer had at least a savings account, or a checking account, or a loan. The number of customers who had a loan is

(a) 11 (b) 38 (c) 37 (d) 50 (e) none of these

5. An exam contains 5 multiple-choice questions, each having 4 possible answers. In how many different ways can the exam be answered? (Assume that every question must be answered.)

(a) 5! (b) $\binom{5}{4}$ (c) 4^5 (d) 5^4 (e) none of these

6. Identify the shaded region in the following Venn diagram:



(a) **(A ∪ B)' ∩ C** (b) **(A ∪ B) ∩ C** (c) **A ∩ B ∩ C**

(d) **(A ∪ B) ∩ C'** (e) **(A ∩ B) ∪ C**

7. If **R** and **S** are finite subsets of a universal set U, such that

n(R') = 20, n(S) = 15, n(S ∪ R') = 30 and n(U) = 35

how many elements are there in **S ∩ R** ?

(a) 20 (b) 15 (c) 5 (d) 0 (e) 10

8. A list of food preferences of 50 species of birds is included when you buy a bird feeder. The list states that 30 species like sunflower seed, 20 like millet, 10 like thistle seed. 10 like both sunflower seed and millet, 5 like both sunflower seed and thistle seed, 4 like both millet and thistle seed. Also, 2 species like all three of the above types of food. How many like none of the above types of food?

(a) 15 (b) 10 (c) 0 (d) 7 (e) 5

9. A set **X** has exactly 6 elements. How many distinct subsets of **X** have two or more elements?

(a) P(6,2) (b) 64 (c) 2^6 - C(6,0) - C(6,1) (d) C(6,2)

(e) 6! - 1! - 0!

10. A poker hand consists of five cards selected from a deck of 52 cards. How many poker hands consist of four hearts and a card from a different suit?

(a) $39 \bullet \binom{13}{4}$ (b) $39 \bullet \binom{52}{4}$ (c) $48 \bullet \binom{13}{4}$

(d) $48 \bullet \binom{52}{4}$ (e) none of these

11. What is the coefficient of x^5y^3 in the expansion of $(x + y)^8$?

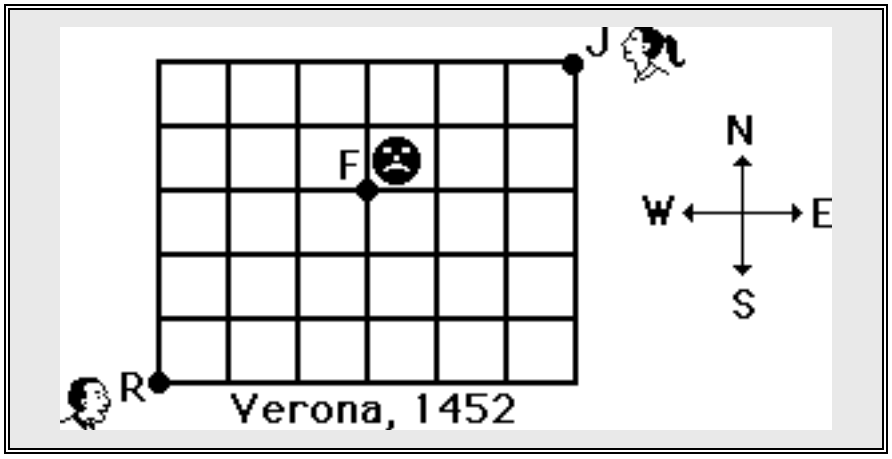
(a) 46 (b) 1 (c) 56 (d) 8 (e) 15

12. A sample (without replacement) of three apples is picked from a bag containing three Red Delicious apples and four Golden Delicious apples. How many such samples consist of exactly two Red Delicious and one Golden Delicious?

(a) $\binom{3}{2}$ (b) $\binom{7}{3} - 4$ (c) $\dfrac{\binom{7}{3}}{2!}$

(d) $\binom{3}{2} \bullet \binom{4}{1}$ (e) $\binom{4}{1}$

13. Here is a street map of Verona in 1452. Romeo is at **R**, Juliet is with her aunt at **J**. The house of Juliet's father is at **F**. In how many ways can Romeo reach Juliet without passing by her father's house, if he travels North and East only?



(a) $\binom{11}{5} - \binom{6}{3} \bullet \binom{5}{2}$ (b) $\binom{11}{5} - 36$ (c) $\binom{9}{3} \bullet \binom{5}{2}$

(d) $\binom{11}{6}$ (e) 5^6

14. Let **S** = {x, y, w, z } be a sample space. Which of the following is a valid probability distribution for **S** ?

(a) Pr(x) = 0.7 Pr(y) = 0 Pr(z) = 0.3 Pr(w) = 0.1

(b) Pr(x) = 0.6 Pr(y) = 0 Pr(z) = 0.3 Pr(w) = 0.1

(c) Pr(x) = 0.5 Pr(y) = 0.1 Pr(z) = –0.3 Pr(w) = 0.1

(d) Pr(x) = 0.6 Pr(y) = 0.1 Pr(z) = 0.2 Pr(w) = 1.1

(e) none of the above

15. How many subsets of the set {1, 2, 3, 4, 5 } do not contain an even digit?

(a) $\binom{5}{2}$ (b) P(5,2) (c) $\frac{5!}{2!}$ (d) 24 (e) 8

16. An experiment consists of observing the color and make of cars in a dealer's lot. Let

E be the event "the car is red"
F be the event "the car is a Chevrolet"
G be the event "the car is green or a Ford"
H be the event "the car is black or green"

Which of the following pair of events are mutually exclusive?

(a) E and F (b) F and G (c) G and H
(d) F and H (e) E and H

17. Suppose a red die and a green die are tossed and the numbers on the uppermost sides are observed. What is the probability that the numbers add up to 4 ?

(a) $\frac{1}{36}$ (b) $\frac{2}{36}$ (c) $\frac{3}{36}$ (d) $\frac{4}{36}$ (e) $\frac{5}{36}$

18. A foundation wishes to award one grant of $ 100,000, two grants of $ 10,000 each, three grants of $ 5,000 each, and three grants of $ 2,000 each. The list of recipients has been already narrowed to 9 recipients. In how many different ways can the awards be made?

(a) 84 (b) 36 (c) 213 (d) 5,040 (e) 72

19. A digit is selected at random from the digits {1, 2, 3, 4, 5, 6, 7, 8, 9 } . What is the probability that the digit is either less than 4 or odd?

(a) $\frac{2}{9}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{4}{9}$ (e) $\frac{4}{3}$

20. See Cover sheet (the one to be handed in) for a description of this question.