

1. Use the normal approximation to the binomial distribution to estimate the probability of getting 55 or more heads in 100 tosses of a fair coin.

- (a) 0.8159      (b) 0.1357      (c) 0.1841      (d) 0.1587      (e) 0.3821

2. If A is a  $3 \times 4$  matrix, and B is a  $4 \times 2$  matrix, then the size of AB is:

- (a)  $2 \times 3$                       (b)  $3 \times 2$                       (c)  $3 \times 4$   
(d)  $4 \times 3$                       (e) none of these

3. Let

$$A = \begin{pmatrix} 3 & 2 & 7 \\ 1 & 0 & 3 \\ 1 & 4 & 2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 5 & 2 \\ 7 & 1 & 3 \end{pmatrix}$$

The third row of  $AB$  is:

- (a) ( 18 23 16 )                      (b) ( 18 16 23 )  
(c) ( 31 11 16 )                      (d) ( 51 20 31 )  
(e) none of these

4. Let  $Z$  be the standard normal random variable. Find a value  $z_0$  so that

$$P(-z_0 \leq Z \leq z_0) = 0.994$$

- (a)  $z_0 = 0.006$                       (b)  $z_0 = 2.5$   
(c)  $z_0 = 2.6$                         (d)  $z_0 = 2.75$   
(e)  $z_0 = -2.6$

5. Yukome Motors produces cars and trucks in three plants, located in **A**tlanta, **B**oston and **C**hicago respectively. The weekly output (measured in hundreds) of 2-door passenger cars, 4-door passenger cars, and trucks at each plant is described by the matrix

$$\begin{array}{l} \text{2-dr} \\ \text{4-dr} \\ \text{truck} \end{array} \begin{pmatrix} \mathbf{At} & \mathbf{Bo} & \mathbf{Ch} \\ 3 & 2 & 2.5 \\ 4 & 4.5 & 3 \\ 5 & 1 & 4 \end{pmatrix} = A$$

The plant in Atlanta operates  $x$  weeks per year, the one in Boston operates  $y$  weeks per year, the one in Chicago operates  $z$  weeks per year. The entry in the second row, first column of the matrix  $A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$  represents the number (in hundreds if appropriate) of

- (a) 2-door passenger cars produced yearly in Chicago.
- (b) cars and trucks produced in Boston.
- (c) 4-door passenger cars produced yearly by Yukome Motors.
- (d) Total production of Yukome Motors.
- (e) none of the above.

6. The probability distribution for a random variable  $X$  is given below. What is the variance of  $X$ ?

$k$	$\Pr(X=k)$
1	0.1
2	0.1
3	0.5
4	0.3

- (a)  $\sqrt{9.8}$
- (b) 6.8
- (c)  $\sqrt{0.8}$
- (d) 9.8
- (e) 0.8

7. The population of scores for a class of 10 students on a 5-question quiz (each question is worth 1 point if answered correctly, 0 otherwise) is shown in the table below. What is the population variance for the scores?

Score	Frequency
0	0
1	1
2	2
3	4
4	2
5	1

- (a)  $\sqrt{1.2}$       (b) 10.2      (c)  $\sqrt{10.2}$       (d) 0      (e) 1.2
8. The amount of regular gasoline sold daily by Tom Quepillo service station is normally distributed with  $\mu = 2,000$  gallons and  $\sigma = 500$  gallons. How many gallons of regular should Mr. Quepillo have on hand at the start of the day in order to be 99.9% sure that he will not run out of regular gasoline before the end of the day?
- (a) 3,650      (b) 3,550      (c) 3,450      (d) 3,350      (e) 2,000

9. Each time a soccer player shoots a penalty kick, her probability of scoring a goal is 0.7 . Let  $X$  be the number of goals she scores out of 5 penalty kicks. What is the standard deviation of  $X$  ?

- (a)  $\sqrt{1.5}$       (b) 1.05      (c)  $\sqrt{1.05}$       (d) 3.5      (e)  $\sqrt{3.5}$

10. If 10% of all pistachios in a large barrel are defective and 400 pistachios are chosen at random from the barrel, estimate the probability that there are only 20 or less defective pistachios among the 400 chosen.

- (a) 0.0548      (b) 0.2      (c) 0.9994      (d) 0.0006      (e) 0.9452

11. Which of the following statements is true about the solution of the following system of equations?

$$\begin{cases} 2x + y = 6 \\ x + 3y = 8 \end{cases}$$

- (a) the value of  $x$  is 2      (b) the value of  $x$  is 1      (c) the value of  $x$  is -2  
(d) there are no solutions      (e) there are infinitely many solutions

12. Let  $X = \begin{pmatrix} 1 & 5 & 1 \\ 2 & 1 & 1 \end{pmatrix}$  and  $Y = \begin{pmatrix} 2 & 5 & 0 & -1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 \end{pmatrix}$

Find the entry in the 2<sup>nd</sup> row and 4<sup>th</sup> column of  $XY$ .

- (a) -2      (b) 5      (c) 0      (d) 1      (e) 7



**THE NEXT TWO QUESTIONS (15 & 16) REFER TO THE FOLLOWING SITUATION:**

The Elfintree Co. makes three types of cookies, **I**, **II** and **III**, each consisting of certain amounts of **Almond paste**, **Butter**, **Chocolate**, and **Dates** (mixed with ten grams of basic cookie dough.) The amounts (in grams) of each ingredient required by each cookie is shown in the matrix **M**:

$$M = \begin{matrix} & \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \\ \mathbf{I} & \left( \begin{array}{cccc} 1 & 0 & 2 & 4 \\ 3 & 2 & 1 & 1 \\ 2 & 5 & 3 & 1 \end{array} \right) \\ \mathbf{II} & & & & \\ \mathbf{III} & & & & \end{matrix}$$

15. The Elfintree Company receives an order for cookies of type **I**, **II** and **III** as shown in the matrix **R** given below. Interpret the entry in the third column, first row of **RM**.

$$R = \begin{matrix} & \mathbf{I} & \mathbf{II} & \mathbf{III} \\ \left( \begin{array}{ccc} 10 & 20 & 15 \end{array} \right) & & & \end{matrix}$$

- (a) The amount (in grams) of chocolate needed to fill the order.
- (b) The total weight (in grams) of cookies ordered.
- (c) The total weight (in grams) of the cookies of type **II**.
- (d) The total weight (in grams) of the cookies of type **III**.
- (e) None of the above.

16. The profit (in cents) which the Elfintree Company makes per cookie of each type is shown in the matrix **P** given below.

$$P = \begin{matrix} \left( \begin{array}{c} 56 \\ 75 \\ 38 \end{array} \right) & \begin{matrix} \mathbf{I} \\ \mathbf{II} \\ \mathbf{III} \end{matrix} \end{matrix}$$

The meaning of the matrix **RP** is:

- (a) Total profit per cookie type
- (b) Total cost of the order
- (c) Total amount of ingredients needed
- (d) Total profit on the order
- (e) **RP** cannot be computed.

17. For certain types of fluorescent lights the amount of hours a bulb will burn before requiring replacement is a random variable with mean  $\mu = 3,000$  hours and  $\sigma = 250$  hours. Suppose that 5,000 such bulbs are installed in the JACC. Estimate the number that will require replacement between 2,000 and 4000 hours from the time of installation. (Use *Tchebyshev's*)

(a)  $\geq \frac{15}{16}$       (b)  $\geq 4,844$       (c)  $\geq 313$       (d)  $\leq 313$       (e)  $\geq 4,688$

18. The solution of the linear system shown below is:

$$\begin{cases} x + 2z = 9 \\ y + z = 1 \\ 3x - 2y = 9 \end{cases}$$

(a)  $x = 1 ; y = 2 ; z = 4$       (b)  $x = 1 ; y = -3 ; z = 4$       (c)  $x = 4 ; y = -3 ; z = 1$   
(d)  $x = -3 ; y = 4 ; z = 1$       (e)  $x = 1 ; y = 3 ; z = -4$

19. A certain device contains three separate components, each of which can fail independently of the other two. The probabilities of failure for each component are 0.1, 0.2 and 0.1 respectively. The device fails only if all three components fail. Estimate the probability that three or more devices fail in a lot of 1,000 such devices chosen at random. (*Exactly one of the answers below is the most accurate.*)
- (a) 36%      (b) 64%      (c) 25%      (d) 75%  
(e) 98.9%

20. See Cover sheet (the one to be handed in) for a description of this question.