1. What is the solution of the following system of linear equations?

$$\begin{cases} x - y + z = 2 \\ x - 2y + 2z = 3 \\ 2x + y - 2z = 2 \end{cases}$$

(a) $x = 1 ; y = -2 ; z = 1$ (b) $x = -1 ; y = 2 ; z = 1$ (c) $x = 1 ; y = 2 ; z = -1$

(d) $x = -1 ; y = -2 ; z = 1$ (e) $x = 1 ; y = -2 ; z = -1$

2. Let A be the 3x3 matrix shown below.

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

The matrix A^3 is:

(a) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$ (e) $\begin{pmatrix} 0 & 3 & 0 \\ 0 & 0 & 3 \\ 3 & 0 & 0 \end{pmatrix}$

3. Let $\mathbf{U} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ be the universe and let

$\mathbf{A} = \{\text{odd integers in U}\}$; $\mathbf{B} = \{1, 2, 3, 4, 5\}$; $\mathbf{C} = \{\text{multiples of 3 in U}\}$

Then the set $\mathbf{(A \cap B') \cup (B \cap C') \cup (C \cap A')}$ is

(a) $\{3\}$ (b) $\{1, 2, 3, 5, 9\}$ (c) $\mathbf{U}$ (d) $\{1, 2, 4, 5, 6, 7, 9\}$

(e) $\varnothing$

4. In 1996 the sun shone in South Bend during 123 days (but it may have also rained during some of those days.). In addition, in 1996 it rained on 168 days in South Bend (but on some of those days the sun also shone.) Finally, in 1996 in South Bend there were 99 days during which it did not rain but it was cloudy all day. During how many days could you have seen a rainbow in South Bend in 1996?

(a) 12 (b) 24 (c) 36 (d) 190 (e) 56

5. A survey of 100 bank customers revealed that 58 of them had a savings account, 63 of them had a checking account, 22 of them had a savings account and a loan, 16 of them had a checking account and a loan, 27 of them have only a checking account, 12 of them had a checking account, a savings account and a loan. Each customer had at least a savings account, or a checking account, or a loan. The number of customers who had a loan is

(a) 11 (b) 38 (c) 37 (d) 50 (e) none of these

6. You have 4 apples, 2 bananas and an orange. In how many different ways can you eat one piece of fruit per day next week?

(a) 5,040 (b) 105 (c) 48 (d) 36 (e) 13,612,578

7. How many possible ways are there of lining up 8 children for a photograph if 2 of the children (Sid and Nancy) refuse to stand next to each other?

(a) $8! - 2 \times 7!$ (b) $\frac{8!}{2}$ (c) $2 \times 7!$ (d) C(8,6) (e) P(8,6)

8. A sample (without replacement) of three apples is picked from a bag containing three Red Delicious apples and four Golden Delicious apples. How many such samples consist of exactly two Red Delicious and one Golden Delicious?

(a) $\binom{3}{2}$ (b) $\binom{7}{3} - 4$ (c) $\frac{\binom{7}{3}}{2!}$

(d) $\binom{3}{2} \bullet \binom{4}{1}$ (e) $\binom{4}{1}$

9. What is the coefficient of p^4p^4 in the binomial expansion of $(p + q)^8$?

(a) 210 (b) 56 (c) 70 (d) 30 (e) 21

10. In how many ways can the 14 children in a third grade class be paired up for a trip to the movies?

(a) $\frac{14!}{7!\text{x}2^7}$ (b) $\frac{14!}{2^7}$ (c) $\binom{14}{2}$ (d) $\binom{14}{7}$ (e) $\frac{14!}{7!\text{x}2!}$

11. Two fair dice are rolled, and the numbers on their top faces are recorded. Consider the following events:

E : both numbers are odd

F : the sum of the two numbers is odd

G : at least one of the two numbers is a 5

Which of the following statements about these events is true?

(a) E and F are mutually exclusive
(b) E and G are mutually exclusive
(c) F and G are mutually exclusive
(d) each pair of these events is mutually exclusive
(e) no two of these events are mutually exclusive

12. Suppose E and F are independent events. Which of the following statements is NOT true?

(a) E and F' are independent
(b) E' and F are independent
(c) E and E' are independent
(d) E' and F' are independent
(e) E and $F \cup F'$ are independent

13. Suppose that E and F are independent events with Pr(E) = 0.3 and Pr(F) = 0.4. What is Pr(E∪F)?

(a) 0.82 (b) 0.7 (c) 0.12 (d) 0.68 (e) 0.58

14. A fair coin is tossed three times. What is the probability of obtaining head three times, given that at least two heads are obtained?

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{1}{8}$ (e) 1

15. A store has fifteen cartons of eggs. Five of the cartons have broken eggs in them. A shopper inspects three cartons, one carton at a time at random (she does not inspect the same carton twice!) and will buy the first good one (no broken eggs) she finds, if any. Otherwise she will not buy any eggs. What is the probability she will not buy any eggs?

(a) $\frac{1}{5}$ (b) $\frac{47}{1264}$ (c) $\frac{1}{3}$ (d) $\frac{2}{91}$ (e) $\frac{2}{21}$

16. The table below gives the distribution of voter registration and voter turnouts for a certain city. A randomly chosen person is questioned at the polls. What is the probability that the person is an Independent?

	Proportion registered	*Proportion turnout*
Democrat	0.50	0.40
Republican	0.20	0.50
Independent	0.30	0.70

(a)0.51 (b) 0.70 (c) 0.21 (d) $\frac{17}{7}$ (e) $\frac{7}{17}$

THE NEXT THREE QUESTIONS (17, 18 and 19) REFER TO THE FOLLOWING SITUATION:

The table below shows the number of 1-headed, 2-headed, 3-headed and 4-headed dragons on the island of Sarkany

1-headed	2-headed	3-headed	4-headed
12	38	38	12

17. What is the expected number of heads per dragon on Sarkany?
(a) 3.8 (b) 3 (c) 1.2 (d) 2 (e) 2.5

18. What is the variance of the number of heads per dragon in the population of dragons on Sarkany?
(a) 6.25 (b) 2.5 (c) 0.73 (d) $\sqrt{2.5}$ (e) 0.43

19. What are the units used to measure the above variance?

(a) $\dfrac{\text{dragons}}{\text{heads}}$ (b) $\left(\dfrac{\text{dragons}}{\text{heads}}\right)^2$ (c) $\sqrt{\text{dragons}}$

(d) $\sqrt{\dfrac{\text{heads}}{\text{dragons}}}$ (e) $\left(\dfrac{\text{heads}}{\text{dragons}}\right)^2$

20. An experiment consists of tossing a fair die 10 times and counting the number of sixes. What is the probability that the count will be 1 or 2?

(a) $\binom{10}{1} + \binom{10}{2}$ (b) $\binom{10}{1}\left(\frac{1}{6}\right)^{1}\left(\frac{5}{6}\right)^{9} + \binom{10}{2}\left(\frac{1}{6}\right)^{2}\left(\frac{5}{6}\right)^{8}$ (c) $\binom{10}{1}\binom{10}{2}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)$

(d) $\left[\binom{10}{1} + \binom{10}{2}\right]\left(\frac{1}{6}\right)^{10}$ (e) 1

21. Buonladro Bakery asserts that the weight of a loaf of Italian bread sold at the bakery is normally distributed with $\mu = 750$ grams and $\sigma = 10$ grams. On three consecutive days the loaves of bread you bought independently at the bakery have weighted less than 740 grams. Mister Buonladro says it's just a coincidence. What is the probability of this "coincidence"? *(Exactly one of the answers below is the most accurate.)*

(a) 0.1587 (b) 10% (c) 0.40% (d) 0.8413 (e) 0.01%

22. An electronic firm determines that the number of defective transistors in each batch of 400 transistors it produces averages 15 defectives, with a standard deviation of 10. Suppose 100 batches are produced. Use Tchebyshev's to estimate the number of batches having between 0 and 30 defective transistors each.

(a) ≤ 0.56 (b) ≥ 56 (c) < 56 (d) ≥ 68 (e) ≥ 112

23. Fesso lottery tickets on the island state of Taxem cost $1.00 each. Winning tickets pay $2.00 (you actually win only one dollar.) The government advertises that, on the average, there are 9 winning tickets among every 19 tickets sold. You buy 100 Fesso lottery tickets. Estimate the probability that you will at least break even.

(a) 0.27 (b) 0.3413 (c) 0.0060 (d) 0.63 (e) $\frac{19}{810}$

24. For certain types of alkaline batteries the amount of hours a battery will last before going dead is a random variable with mean $\mu = 3{,}000$ hours and $\sigma = 250$ hours. Suppose that 5,000 new such batteries are purchased by Notre Dame to illuminate An Tostal at night. Estimate the number that will go dead between 2,000 and 4000 hours from the time of purchase.

(a) $\geq \frac{15}{16}$ (b) $\geq 4{,}844$ (c) ≥ 313 (d) ≤ 313 (e) $\geq 4{,}688$

25. The Borrachín restaurant serves only Chianti wine to its customers. The amount of Chianti wine sold daily by the Borrachín restaurant is normally distributed with $\mu = 200$ bottles and $\sigma = 20$ bottles. What is the smallest number of bottles of Chianti which the Borrachín's owner should have on hand at the start of the day, in order to be 99.9% sure that he will not run out of wine before the end of the day?

(a) 408 (b) 510 (c) 225 (d) 262 (e) 275

26. A clothing manufacturer has factories in Los Angeles, San Antonio and Newark. The sales (in thousands of units) for last year are given by the table below.

	Los Angeles	San Antonio	Newark
Coats	12	13	38
Shirts	25	5	26
Sweaters	11	8	8
Ties	5	0	12

Prices for coats, shirts, sweaters and ties last year were
$ 100 $ 10 $ 25 $ 5
respectively. The total revenue produced by the Los Angeles factory, the San Antonio factory and the Newark factory, in that order, is given by:

(a) [1,750 1,550 4,320]

(b) [1,200 250 275 25]

(c) [2,215 1,750 3,250]

(d) [1,650 1500 4,320]

(e) [1,550 1,750 4,320]

27. Which of the following is **NOT** a stochastic matrix?

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0.3 & 0.6 & 0.1 \\ 0.2 & 0.3 & 0.5 \\ 0.5 & 0.1 & 0.4 \end{bmatrix}$

(d) $\begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0.6 & 0.3 & 0.1 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}$

(e) $\begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 1 \end{bmatrix}$

28. Which of the following is a regular stochastic matrix?

(a) $\begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.8 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0.2 \\ 1 & 0.8 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 0.2 & 0.8 \\ 0.3 & 0.7 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

29. The changes in weather from day to day on the planet Xantar form a regular Markov process. Each day is either rainy or sunny, except that on Christmas Day it is always sunny, by order of the High, Exalted, All Wise, All Knowing King of Xantar. If it rains one day, there is a 90% chance that it will be sunny the following day. If it is sunny one day, there is a 60% chance of rain the next day. What is the probability of rain on December 24 on Xantar?

(a) 40% (b) 50% (c) 60% (d) 45% (e) 25%

30. See cover page.