Formulas that you might want to use.

1. If A is an absorbing stochastic matrix with

$$A = \left[\begin{array}{c|c} I & S \\ \hline 0 & R \end{array}\right]$$

then the stable matrix of A is

$$\left[\begin{array}{c|c} I & S(I-R)^{-1} \\ \hline 0 & 0 \end{array}\right]$$

where the identity matrix I in $(I-R)^{-1}$ is chosen to be the same size as R.

2. Compound Interest.

Compound amount $F = (1 + i)^n P$

Present value $P = \dfrac{F}{(1 + i)^n}$

3. Simple interest.

Amount $A = (1 + nr)P$

4. Annuities.

$$F = S_{n\,i} R, \quad S_{n\,i} = \frac{(1 + i)^n - 1}{i}$$

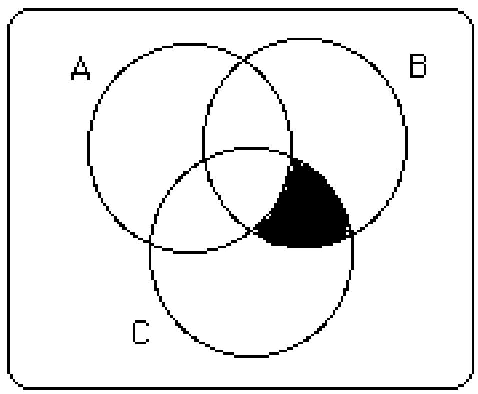
$$P = a_{n\,i} R, \quad a_{n\,i} = \frac{(1 + i)^n - 1}{i\,(1 + i)^n}$$

MATH 104 - FINAL EXAM

1. In a 10-team soccer conference, each team plays every other team exactly once. How many games must be played?

   (a) 45 (b) 90 (c) 99 (d) 100 (e) 10!

2. Identify the following shaded region.



   (a) $(A' \cup B) \cap C$ (b) $A' \cap B \cap C$ (c) $(A' \cap B) \cup (A' \cap C)$
   (d) $A' \cup B \cup C$ (e) none of the above

3. Suppose that, in a certain experiment, the events E and F are independent. If $Pr(E) = Pr(F) = \frac{1}{2}$, what is $Pr(E \cup F)$?

   (a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{7}{8}$

(e) not enough information

4. Suppose that E and F are events in an experiment, and $\Pr(E) = \frac{1}{4}$, $\Pr(F) = \frac{1}{2}$, $\Pr(E \cup F) = \frac{3}{4}$. What is $\Pr(E|F)$?

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 0 (e) $\frac{1}{3}$

5. A random variable X has the following probability distribution.

k	Pr(X = k)
-10	1/3
0	1/3
1	1/6
2	1/6

What is the expected value E(X)?

(a) $\frac{-7}{3}$ (b) $-\frac{17}{6}$ (c) $\frac{-7}{6}$ (d) $\frac{1}{3}$ (e) $\frac{1}{6}$

6. An urn contains 3 red and 5 green balls. Three balls are drawn without replacement. What is the probability that the three balls have the same color?

(a) $\frac{11}{56}$ (b) $\frac{20}{56}$ (c) $\frac{13}{56}$ (d) $\frac{15}{56}$ (e) $\frac{21}{56}$

7. A random variable X has the following probability distribution

k	Pr(X = k)
-12	1/6
0	1/3
2	1/2

Find the standard deviation, σ, of X.

(a) 4 (b) -1 (c) 5 (d) $\sqrt{\frac{74}{3}}$ (e) 6

8. Find y such that the table below represents a possible probability distribution of a random variable X.

k	Pr(X = k)
0	.1
1	.2
2	.4
3	y

(a) .6 (b) .3 (c) .1 (d) .2 (e) 0

9. An Olympic pistol shooter has $\frac{2}{3}$ chances of hitting the target at each shot. Find the probability that he will hit exactly 10 targets in a game of 15 shots.

(a) $1-\binom{15}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{5}$ (b) $\binom{15}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{5}$ (c) $\left(\frac{2}{3}\right)^{10}$

(d) $\binom{15}{10}\left(\frac{2}{3}\right)^{5}\left(\frac{1}{3}\right)^{10}$ (e) $1-\left(\frac{1}{3}\right)^{5}$

10. The amount of milk contained in a gallon container in normally distributed with mean 128.2 ounces and standard deviation 0.2 ounces. What is the probability that a random bottle contains less than 128 ounces?

(a) 0.3085 (b) 0.8413 (c) 0.1587

(d) 0.6915 (e) 0.5

11. A dice is rolled 180 times. Use the normal approximation to estimate the probability of getting at least 27 fives.

(a) 0.2743 (b) 0.7257 (c) 0.242

(d) 0.758 (e) 0.6915

12. Find the area under the standard normal curve to the right of $z = \frac{1}{2}$. (Use the attached table.)

(a) 0.5000 (b) 0.6915 (c) -0.7 (d) 0.3085

(e) 0.2500

13. A fair die is rolled three times, what are the odds in favor of obtaining different numbers on the top face in all three rolls?

(a) 4 to 1 (b) 4 to 5 (c) 5 to 4 (d) 9 to 4

(e) 4 to 9

14. What is the equation of a line passing through the point (1,2) and parallel to the line $3x - y = 1$

(a) $y = -\frac{1}{3}x$ (b) $y = -\frac{1}{3}x + \frac{7}{3}$ (c) $y = 3x - 5$

(d) $y = 3x$ (e) $y = 3x - 1$

15. Given that $\begin{bmatrix} 4 & -2 & 3 \\ 8 & -3 & 5 \\ 7 & -2 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -2 & 2 & -1 \\ 3 & -5 & 4 \\ 5 & -6 & 4 \end{bmatrix}$

Solve for z in the following system of equations:

$$4x - 2y + 3z = 1$$
$$8x - 3y + 5z = 1$$
$$7x - 2y + 4z = 1$$

(a) $z = -1$ (b) $z = 1$ (c) $z = 9$ (d) $z = 3$

(e) $z = 2$

16. Which of the following statements is true about the solution to the system of equations given below?

$$\begin{aligned} x + y + z &= 4 \\ y + 2z &= 2 \\ x + z &= 2 \end{aligned}$$

(a) $z = 4$ (b) $z = 0$ (c) $z = 2$ (d) $z = 1$
(e) $z = -1$

17. Use the Gauss Jordan method to find the inverse of the matrix
$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$.

(a) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $A^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ 2 & -1 & -1 \\ -2 & 0 & 1 \end{bmatrix}$

(c) $A^{-1} = \begin{bmatrix} -1 & 1 & 0 \\ -2 & 1 & -1 \\ 2 & 0 & -1 \end{bmatrix}$ (d) $A^{-1} = \begin{bmatrix} -1 & -1 & -1 \\ 0 & -1 & -1 \\ -2 & -2 & -1 \end{bmatrix}$

(e) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 1 & 1 \\ 2 & 0 & -1 \end{bmatrix}$

18. Find the product

$$\begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}.$$

(a) $\begin{bmatrix} 2 & 3 \\ 2 & 4 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 2 & 2 \\ 3 & 4 & 3 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 1 & 2 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \end{bmatrix}$

19. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix}$.
Find the entry in the 2^{nd} row and 2^{nd} column of A^{-1}.

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 0 (d) 1 (e) –1

20. Peter either jogs or swims (but not both) each day. If he jogs one day there is a 90% chance that he will swim the following day, whereas if he swims one day there is only a 70% chance that he will swim the following day also. If Peter jogs on Monday, what is the probability that he will jog on Wednesday?

(a) .1 (b) .72 (c) .28 (d) .9 (e) .3

21. The transition matrix of a Markov Process is given by the matrix. $\begin{bmatrix} .8 & .3 \\ .2 & .7 \end{bmatrix}$. The stable distribution of this process is:

(a) $\begin{bmatrix} \frac{2}{5} \\ \frac{3}{5} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$ (c) $\begin{bmatrix} \frac{3}{5} \\ \frac{2}{5} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{4}{5} \\ \frac{1}{5} \end{bmatrix}$ (e) $\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$

22. Consider the following matrices:

$$X = \begin{bmatrix} \frac{1}{2} & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \qquad Y = \begin{bmatrix} \frac{1}{3} & \frac{1}{4} & 0 \\ \frac{2}{3} & \frac{1}{4} & 0 \\ 0 & \frac{1}{4} & 1 \end{bmatrix} \qquad Z = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{2} \end{bmatrix}$$

Which of these matrices are regular stochastic matrices?

(a) Z and X only (b) Z only (c) Z and Y only
(d) X only (e) X and Y only

23. Find the stable matrix of the absorbing stochastic matrix

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{4} \\ 0 & 0 & \frac{1}{4} \end{bmatrix}.$$

(a) $\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{4} \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 & \frac{2}{3} \\ 0 & 1 & \frac{1}{3} \\ 0 & 0 & \frac{3}{4} \end{bmatrix}$

24. Let $T = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{3}{4} \end{bmatrix}$ be the transition matrix of a Markov Process. If the distribution of the current generation is $\begin{bmatrix} \frac{2}{5} \\ \frac{3}{5} \end{bmatrix}$, find the distribution of the next generation.

(a) $\begin{bmatrix} \frac{1}{4} \\ \frac{3}{4} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

(c) $\begin{bmatrix} \frac{7}{20} \\ \frac{13}{20} \end{bmatrix}$

(d) $\begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}$

(e) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

25. The transition matrix of a Markov process is given by the matrix

$$A = \begin{bmatrix} .1 & .2 & 0 \\ .8 & .3 & 0 \\ .1 & .5 & 1 \end{bmatrix}$$

Which of the following statements is true?

(a) A is regular (b) A is absorbing (c) A is regular and absorbing

(d) A is neither regular nor absorbing (e) none of the above

26. Alice deposits $1,000 in an account paying 12% annual interest compounded semiannually. How much will be in the account five years from now?

(a) $1,790.85 (b) $1,558.39 (c) $1,600.00

(d) $1,420.50 (e) $1,950.22

27. Betty decides to take a year off work (without pay) and travel around the world. She has $40,000 in her bank account at the beginning of the year. The account pays 6% annual interest compounded monthly. Betty wants to withdraw a fixed amount each month leaving a balance of zero at the end of the year, how much should be withdrawn each month?

(a) $3,000.00 (b) $3,242.66 (c) $3,333.33

(d) $3,442.66 (e) $3,500.20

28. Catherine needs $10,000 five years from now in order to pay off a loan. How much should she save each month for the next five years if annual interest rates are 9% compounded monthly?

(a) $200.22 (b) $166.67 (c) $207.58

(d) $150.28 (e) $132.58

29. Delilah took out a 30 year mortgage for $80,000 to purchase a condo. The annual interest rate is 12%, compounded monthly, with payments made monthly. What is the monthly payment?

(a) $831.22 (b) $22.89 (c) $822.89

(d) $751.22 (e) $632.50

30. Each month Elvira deposits $100 in a savings account receiving 6% annual interest compounded monthly. How much will Elvira have in the account at the end of 4 years?

(a) $5,409.78 (b) $4,800.00 (c) $4,258.03

(d) $4,961.22 (e) $5,223.50